Spacecraft Dynamics and Control

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Lecture 11: Intro to Rocketry

Introduction

In this Lecture, you will learn:

Introduction to Rocketry

- Mass Consumption
- Specific Impulse and Rocket Types
- Δv limitations
- Staging

Numerical Problem: Suppose our mission requires a dry weight of 30kg. How much propellant is required to achieve a circular orbit of altitude $200km$?

Questions about Propulsion

We have talked a bit about Δv.

- How is Δv created?
- How expensive is it?
- Is it really instantaneous?

Δv budget

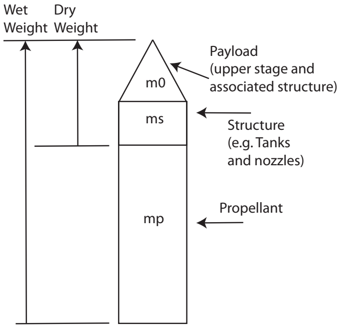
A Typical mission uses a lot of Δv.

- How much propellant will we need?
- What is the maximum Δv budget?

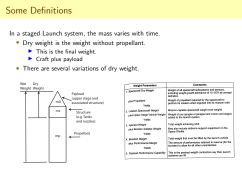
Propulsion Function	Typical Requirement
Orbit transfer to GEO (orbit insertion)	
• Perigee burn	2,400 m/s
• Apogee burn	1,500 (low inclination) to 1,800 m/s (high inclination)
Initial spinup	1 to 60 rpm
LEO to higher orbit raising ΔV	60 to 1,500 m/s
• Drag-makeup ΔV	60 to 500 m/s
• Controlled-reentry ΔV	120 to 150 m/s
Acceleration to escape velocity from LEO parking orbit	3,600 to 4,000 m/s into planetary trajectory
On-orbit operations (orbit maintenance)	
• Despin	60 to 0 rpm
• Spin control	±1 to ±5 rpm
• Orbit correction ΔV	15 to 75 m/s per year
• East-West stationkeeping ΔV	3 to 6 m/s per year
• North-South stationkeeping ΔV	45 to 55 m/s per year
• Survivability or evasive maneuvers (highly variable) ΔV	150 to 4,600 m/s
Attitude control	3–10% of total propellant mass
• Acquisition of Sun, Earth, Star	Low total impulse, typically <5,000 N•s, 1 K to 10 K pulses, 0.01 to 5.0 sec pulse width
• On-orbit normal mode control with 3-axis stabilization, limit cycle	100 K to 200 K pulses, minimum impulse bit of 0.01 N•s, 0.01 to 0.25 sec pulse width
• Precession control (spinners only)	Low total impulse, typically <7,000 N•s, 1 K to 10 K pulses, 0.02 to 0.20 sec pulse width
• Momentum management (wheel unloading)	5 to 10 pulse trains every few days, 0.02 to 0.10 sec pulse width
• 3-axis control during ΔV	On/off pulsing, 10 K to 100 K pulses, 0.05 to 0.20 sec pulse width

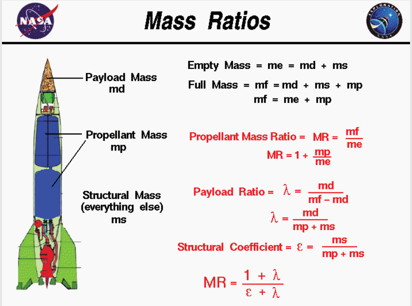
In a staged Launch system, the mass varies with time.

- Dry weight is the weight without propellant.
  - This is the final weight.
  - Craft plus payload
- There are several variations of dry weight.



Weight Parameters	Comments
1. *Spacecraft Dry Weight*	Weight of all spacecraft subsystems and sensors, including weight growth allowance of 15–25% at concept definition
plus Propellant	Weight of propellant required by the spacecraft to perform its mission when injected into its mission orbit
Yields	
2. *Loaded Spacecraft Weight*	Mission-capable spacecraft weight (wet weight)
plus Upper Stage Vehicle Weight	Weight of any apogee or perigee kick motors and stages added to the launch system
Yields	
3. *Injected Weight*	Total weight achieving orbit
plus Booster Adapter Weight	May also include airborne support equipment on the Space Shuttle
Yields	
4. *Boosted Weight*	Total weight that must be lifted by the launch vehicle
plus Performance Margin	The amount of performance retained in reserve (for the booster) to allow for all other uncertainties.
Yields	
5. *Payload Performance Capability*	This is the payload weight contractors say their launch systems can lift





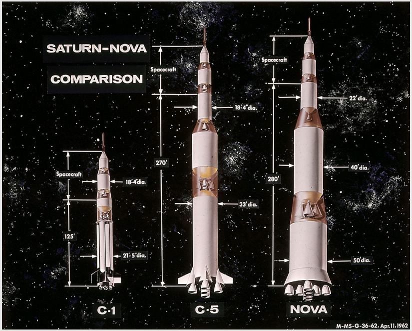


figure from NASA

How to create Thrust: Newton's Second Law

Approximation: Consider the expulsion of a piece of propellant, Δm.

Initial State:

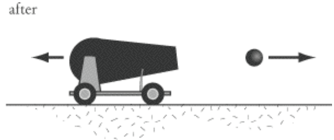
- Propellant and Rocket move together.
- Total Momentum:

$$h_i = (m_r + \Delta m)v$$

Final State:

- Propellant and Rocket move separately.
- Rocket has velocity $v + \Delta v$
- Propellant has velocity $v - c$.
  - c is the exhaust velocity
- Total Momentum:

$$h_f = m_r(v + \Delta v) + \Delta m(v - c)$$



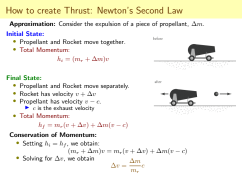
Conservation of Momentum:

- Setting $h_i = h_f$, we obtain:

$$(m_r + \Delta m)v = m_r(v + \Delta v) + \Delta m(v - c)$$

- Solving for Δv, we obtain

$$\Delta v = \frac{\Delta m}{m_r} c$$



- In this slide, h_i is the initial linear momentum of rocket and propellant mass
- v is the initial velocity of the spacecraft
- m_r is the mass of the rocket
- Δm is the mass of the propellant.
- h_f is the final linear momentum of the rocket combined with the propellant. By conservation of momentum, $h_i = h_f$
- Δv is the change in velocity of the rocket.
- **Note:** This is for a single particle of propellant - Δm and can not be used to calculate Δv directly. We will integrate this equation of many particles of propellant to get the true Δv.

Continuous Thrust: The Rocket Equation

For a single particle of propellant, we have

$$\Delta v = \frac{\Delta m}{m_r} c$$

Dividing by Δt and taking the limit as $\Delta t \to 0$, we get

$$\dot{v}(t) = \frac{\dot{m}_r(t)}{m_r(t)} c$$

where we often assume constant mass flow rate $\dot{m}_r(t)$.

Returning to the differential form, we can directly integrate

$$dv = \frac{c}{m_r} dm_r$$

to obtain the Rocket Equation:

$$\Delta v = v(t_f) - v(t_0) = c \ln \left[\frac{m(t_0)}{m(t_f)} \right]$$

Which is quite different from the approximation $\Delta v = \frac{\Delta m}{m_r} c$!

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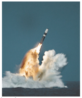

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Which is quite different from the approximation $\Delta v = \frac{\Delta m}{m_r} c$!

- $\dot{m}_r = \lim_{\Delta t \to 0} \frac{\Delta m}{\Delta t}$ is the rate at which we are using up propellant. Note this doesn't affect Δv!
- Although liquid and hybrid rockets can control this rate, in practice, we want to make it as large as possible so that the Δv happens quickly.
- $m(t_0)$ is the mass before the burn. $m(t_f)$ is the mass after the burn.
- $v(t_f)$ is the velocity after the burn. $v(t_0)$ is the velocity before the burn.

Sizing the Propellant: Inverse Rocket Equation

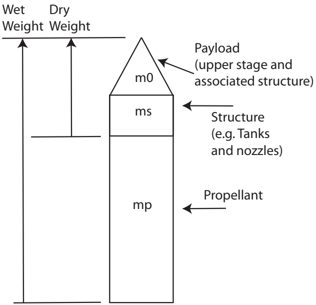
Now we have an expression for Δv as a function of wet and dry weights.

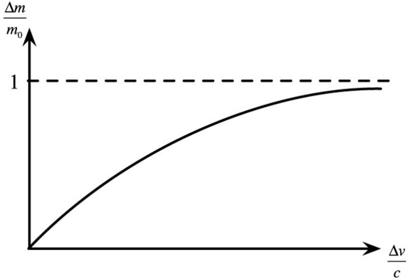
$$\Delta v = v(t_f) - v(t_0) = c \ln \left[\frac{m(t_0)}{m(t_f)} \right]$$

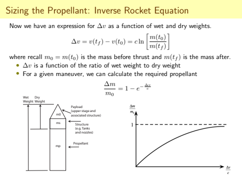
where recall $m_0 = m(t_0)$ is the mass before thrust and $m(t_f)$ is the mass after.

- Δv is a function of the ratio of wet weight to dry weight
- For a given maneuver, we can calculate the required propellant

$$\frac{\Delta m}{m_0} = 1 - e^{-\frac{\Delta v}{c}}$$





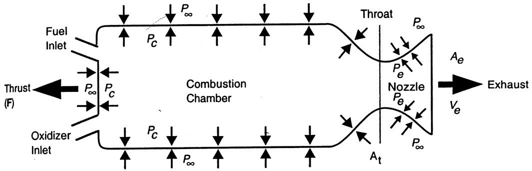


- $m(t_0)$ is the wet weight (with propellant).
- $m(t_f)$ is the dry weight (after all propellant has been used up)
- First equation is called the rocket equation (May 10, 1897), derived by Konstantin E. Tsiolkovsky (1857-1935). Recluse who lived in a log cabin outside Moscow. First person to conceive of space elevator (inspired by Eiffel tower).
- Assuming no structural mass, to launch a 2000 kg rocket to Alpha Centaur would require a rocket of more than 10,000,000 kg (weight of Eiffel Tower) and take 142,000 years to arrive.

Effective Exhaust Velocity

The efficiency of the rocket depends on the relative velocity of the propellant, c.

- However, there is also a force due to pressure, $F = A_e(P_e - P_\infty)$.

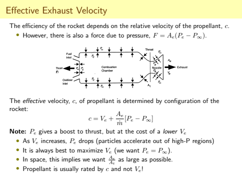


The *effective* velocity, c, of propellant is determined by configuration of the rocket:

$$c = V_e + \frac{A_e}{\dot{m}}[P_e - P_\infty]$$

Note: P_e gives a boost to thrust, but at the cost of a *lower* V_e

- As V_e increases, P_e drops (particles accelerate out of high-P regions)
- It is always best to maximize V_e (we want $P_e = P_\infty$).
- In space, this implies we want $\frac{A_e}{A_t}$ as large as possible.
- Propellant is usually rated by c and not V_e!



- In previous slides, we have been using c instead of V_e for exhaust velocity so as not to confuse you later. However, these formulae should be used with *effective* exhaust velocity, c.
- P_∞ is the atmospheric pressure.
- P_e is the pressure at exit from the nozzle.
- A_t is the area of the throat.
- A_e is the area of the nozzle exit.
- The effective exhaust velocity of H_2-O_2 propellant in space is 4,440 m/s
- On the ground, there is an optimal A_e corresponding to $P_e = P_\infty$.
- Expansion ratio (A_e/A_t) of 117:1 for Merlin 1D Falcon Heavy upper stage.

Pressure Changes affect Efficiency on Saturn V

Specific Impulse

Definition 1.

The **Specific Impulse** is the ratio of the momentum imparted to the weight (on earth) of the propellant.

$$I_{sp} = \frac{\Delta mc}{\Delta mg} = \frac{c}{g}$$

Since $\Delta v = c \ln \left[\frac{m_0}{m_f} \right]$, specific impulse gives a measure of how efficient the propellant is.

Propulsion Technology	Orbit Insertion		Orbit Maintenance and Maneuvering	Attitude Control	Typical Steady State I_{sp} (s)
	Perigee	Apogee			
Cold Gas			✔	✔	30–70
Solid	✔	✔			280–300
Liquid					
Monopropellant			✔	✔	220–240
Bipropellant	✔	✔	✔	✔	305–310
Dual mode	✔	✔	✔	✔	313–322
Hybrid	✔	✔	✔		250–340
Electric		✔	✔		300–3,000

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Dual mode	✓	✓	✓	✓	313–322
Hybrid	✓	✓	✓		250–340
Electric		✓	✓		300–3,000

- measured in seconds, I_{sp} tells, for any amount of propellant mass, how many seconds the rocket will provide thrust equal to the weight ($g = 9.81$) of the propellant consumed.
- Because the effective velocity depends on atmospheric pressure, I_{sp} is different on the surface of the earth vs. in space.
- Typically, I_{sp} assumes a perfectly expanded rocket.
- Isp for Starship is 320 (atmo) to 360 (space) for Oxygen-Methane

Example

Problem: Suppose our mission requires a dry weight of $m_L = 30$kg. Using an I_{sp} of $300s$, how much propellant is required to achieve a circular orbit of altitude $200km$?

Solution: A Circular orbit at 200km requires a total velocity of

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398600}{6578}} = 7.78km/s$$

Add $1.72km/s$ to account for gravity and drag. This totals $9.5km/s = 9500m/s$. The $I_{sp} = 300s$, which means $c = 3000m/s$. Thus we have

$$\frac{m_p}{m_0} = 1 - e^{-\frac{\Delta v}{c}} = .9579$$

Since $m_0 = m_L + m_p$, $m_p = m_L \left(\frac{.9579}{1 - .9579} \right) = 682$kg.

- Which is sort of a lot!
- What about structural mass and Δv for orbital maneuvers?
- We'll return to this problem later

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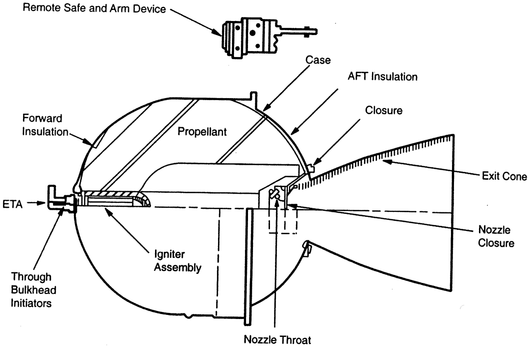
- m_0 is wet mass, dry mass plus propellant.
- $\Delta m = m_p$

Solid Rocket Motors



Advantages:

- Simple
- Reliable
- Low Cost

Disadvantages:

- Limited Performance
- Not Adjustable (Safety)
- Toxic Byproducts

Solid Rocket Motors

Motor	Total Impulse (N·s)	Loaded Weight (kg)	Propellant Mass Fraction	Avg. Thrust (lb_f)	Avg. Thrust (N)	Max. Thrust (N)	Effective I_{sp} (sec)	Status
IUS SRM-1 (ORBUS-21)	2.81×10^7	10,374	0.94	44,610	198,435	260,488	295.5	Flown
LEASAT PKM	9.26×10^6	3,658	0.91	35,375	157,356	193,200	285.4	Flown
STAR 48A	6.78×10^6	2,559	0.95	17,900	79,623	100,085	283.9	Flown
STAR 48B(S)	5.67×10^6	2,135	0.95	14,845	66,034	70,504	286.2	Qualified
STAR 48B(L)	5.79×10^6	2,141	0.95	15,160	67,435	72,017	292.2	Qualified
STAR 62	7.12×10^6	2,459					293.5	In develop.
STAR 75	2.13×10^7	8,066	0.93	44,608	198,426	242,846	288.0	In develop.
IUS SRM-2 (ORBUS-6)	8.11×10^6	2,995	0.91	18,020	80,157	111,072	303.8	Flown
STAR 13B	1.16×10^5	47	0.88	1,577	7,015	9,608	285.7	Flown
STAR 30BP	1.46×10^6	543	0.94	5,960	26,511	32,027	292.0	Flown
STAR 30C	1.65×10^6	626	0.95	7,140	31,760	37,031	284.6	Flown
STAR 30E	1.78×10^6	667	0.94	7,910	35,185	40,990	289.2	Flown
STAR 37F	3.02×10^6	1,149	0.94	9,911	44,086	49,153	291.0	Flown

Figure: Thiokol (ATK Launch Systems) = STAR, LEASAT; United Technologies = IUS

Liquid Monopropellants

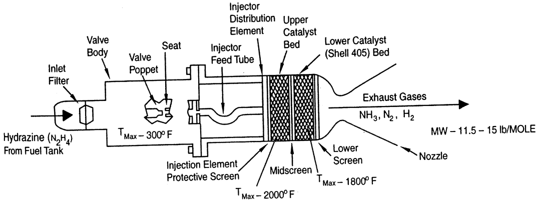


Figure: Typical Hydrazine Monopropellant

Advantages:

- Simple
- Reliable
- Low Cost

Disadvantages:

- Lower Performance than bipropellant

Liquid Bipropellants

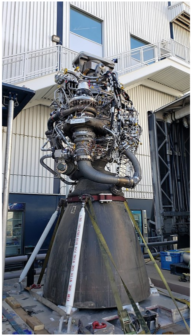


Figure: Raptor Engine

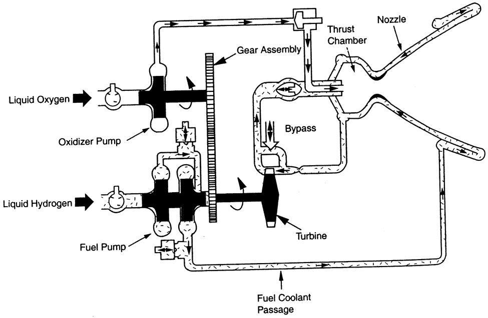


Figure: Centaur O_2-H_2 upper stage.

Advantages:

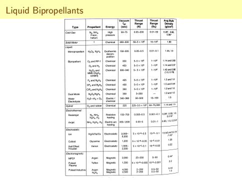
- High Performance
- Adjustable

Disadvantages:

- Complicated
- Dangerous
- Sometimes Toxic

Liquid Bipropellants

Type	Propellant	Energy	Vacuum I_{sp} (sec)	Thrust Range (N)	Thrust Range (lb_f)	Avg Bulk Density (g/cm^3)
Cold Gas	N_2, NH_3, Freon, helium	High pressure	50–75	0.05–200	0.01–50	0.28*, 0.60, 0.96*
Solid Motor	†	Chemical	280–300	$50–5 \times 10^6$	$10–10^6$	1.80
Liquid:						
Monopropellant	H_2O_2, N_2H_4	Exothermic decom-position	150–225	0.05–0.5	0.01–0.1	1.44, 1.0
Bipropellant	O_2 and RP-1	Chemical	350	$5–5 \times 10^6$	$1–10^6$	1.14 and 0.80
	O_2 and H_2	Chemical	450	$5–5 \times 10^6$	$1–10^6$	1.14 and 0.07
	N_2O_4 and MMH (N_2H_4, UDMH)	Chemical	300–340	$5–5 \times 10^6$	$1–10^6$	1.43 and 0.86 (1.0, 0.79)
	F_2 and N_2H_4	Chemical	425	$5–5 \times 10^6$	$1–10^6$	1.5 and 1.0
	OF_2 and B_2H_6	Chemical	430	$5–5 \times 10^6$	$1–10^6$	1.5 and 0.44
	ClF_5 and N_2H_4	Chemical	350	$5–5 \times 10^6$	$1–10^6$	1.9 and 1.0
Dual Mode	N_2O_4/N_2H_4	Chemical	330	3–200	—	1.9 and 1.0
Water Electrolysis	$H_20 \rightarrow H_2 + O_2$	Electric / chemical	340–380	50–500	10–100	1.0
Hybrid	O_2 and rubber	Chemical	225	$225–3.5 \times 10^5$	50–75,000	1.14 and 1.5
Electrothermal:						
Resistojet	N_2, NH_3, N_2H_4, H_2	Resistive heating	150–700	0.005–0.5	0.001–0.1	0.28*, 0.60, 1.0, 0.019*
Arcjet	NH_3, N_2H_4, H_2	Electric arc heating	450–1,500	0.05–5	0.01–1	0.60, 1.0, 0.019*
Electrostatic:						
Ion	Hg/A/Xe/Cs	Electrostatic	2,000–6,000	$5 \times 10^{-6}–0.5$	$10^{-6}–0.1$	13.5/0.44*/2.73*/1.87
Colloid	Glycerine	Electrostatic	1,200	$5 \times 10^{-6}–0.05$	$10^{-6}–0.01$	1.26
Hall Effect Thruster	Xenon	Electrostatic	1,500–2,500	$5 \times 10^{-6}–0.1$	$10^{-6}–0.02$	0.22
Electromagnetic:						
MPD‡	Argon	Magnetic	2,000	25–200	5–50	0.44*
Pulsed Plasma	Teflon	Magnetic	1,500	$5 \times 10^{-6}–0.005$	$10^{-6}–0.001$	2.2
Pulsed Inductive	Argon N_2H_4	Magnetic Magnetic	4,000 2,500	2–200 2–200	0.5–50 0.5–50	0.44 1.0



Impulse Densities of Several Propellants

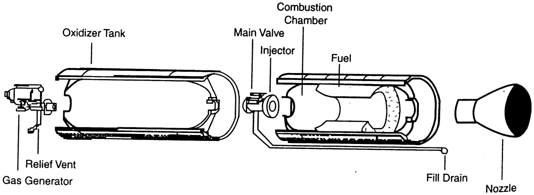
Pc = 1000 PSI to vacuum with 100:1 nozzle

(sorted highest to lowest)

Oxidizer	Fuel	OF Ratio	Pressure (PSI)	Isp(v) (s)	Oxidizer Density lb/cuft	Fuel Density lb/cuft	Avg Density lb/cuft	Density Impulse (lb-f-s/cuft)
AP	HTPB-Al	5.17	1000	312.6	121.7	167.6	127.4	39810
Nitric Acid	Furfuryl Alcohol	2.40	1000	323.0	93.7	70.5	85.5	27600
H2O2 (100%)	Kerosene	7.00	1000	331.0	90.5	49.9	82.2	27195
N2O4	Hydrazine	1.08	1000	348.0	90.1	63.7	75.2	26152
Nitric Acid	Kerosene	4.60	1000	310.0	93.7	49.9	81.0	25111
H2O2 (90%)	Kerosene	7.00	1000	310.0	86.6	49.9	79.3	24587
AK27	T185	3.56	1000	312.0	92.4	49.3	77.5	24191
Lox	Kerosene	2.33	1000	347.0	71.2	49.9	63.1	21899
Lox	IPA	1.70	1000	341.0	71.2	49.1	61.0	20810
AP	HTPB	2.33	1000	224.0	121.7	57.4	91.1	20399
Lox	Butane	2.20	1000	365.0	71.2	37.5	55.6	20290
Lox	Methane	2.77	1000	365.0	71.2	29.0	51.4	18759
Lox	Propane	2.20	1000	355.0	71.2	30.8	50.5	17939
Lox	LH2	6.00	1000	457.0	71.2	4.4	22.6	10307

methalox is 322-365 Isp at average bulk density of .46. But less soot and doesn’t freeze.

Hybrid Rockets



Advantages:

- Throttled
- Non-Explosive

Disadvantages:

- Requires Oxidizer
- Bulky

Flown on SpaceShipOne (Developed by SpaceDev, Oxidizer - N02, $I_{sp} = 250s$, Max Thrust 74kN)

Hybrid Rockets

Motor	Average Thrust (lb_f)	Average Thrust (kN)	Burn Duration (sec)	Fuel	Oxidizer	Comments
American Rocket Company						
H-500	75,000	333	70	HTPB	LOx	Qualified for flight
H-250	32,000	142		HTPB	LOx	In development
H-50	10,000	44		HTPB	LOx	In development
U-50	6,500	29		HTPB	LOx	In development
U-1	100	0.44		HTPB	LOx	In development
United Technologies						
	40,000	178	300	HTPB	IRFNA	Flown on Firebolt air-launched target drone, 1968
StarsTruck						
	40,000	178		CTBN	LOx	Flown on Dolphin water-launched sounding rocket, 1984
USAF Academy						
H-1	55	0.25	2.3	HTPB	GOx	Flown on 4-ft tall rocket for student project, 1991

Figure: American Rocket Company = SpaceDev

Hybrid Rockets

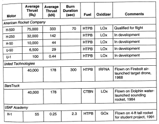


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Figure: American Rocket Company = SpaceDev

- HTPB is a common plastic/rubber hybrid. Also used occasionally in solid rockets
- Commercial sources include paraffin and spandex.

Electric Propulsion

Electrothermal:

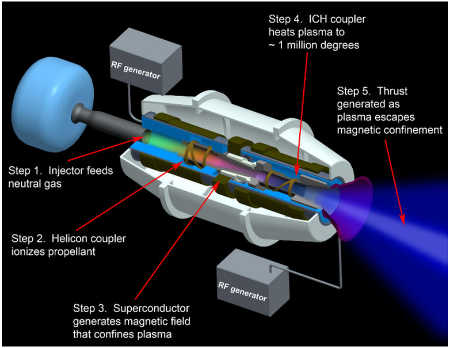
- Ohmic Heating

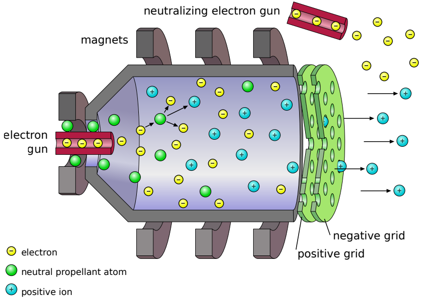
Electrostatic:

- Repulsion/Attraction

Electromagnetic:

- Ions accelerated by EM waves





Advantages:

- Very High Performance

Disadvantages:

- Low Thrust
- Requires Electricity

Electric Propulsion

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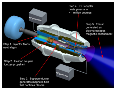

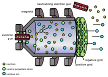

Advantages:
- Very High Performance

Disadvantages:
- Low Thrust
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On left is a magnetoplasmadynamic thruster (MPD)

- Requires MW power for radio heating and magnetic confinement
- Requires a nuclear reactor for power.
- Only experimental MPDs have flown to date.

On right is a gridded ion thruster

- The US in the 1960s focused on GITs, while the Soviet union focused on Hall Effect Thrusters (HET)
- Common for stationkeeping (in GEO)
- Isp in the range 3k-10k (maybe 21k)

Electric Propulsion

The choice of Electrothermal/Electrostatic/Electromagnetic depends on available power.

Electrothermal	Electrostatic	Electromagnetic
• Gas heated via resistance element or arc and expanded through nozzle • Resistojets • Arcjets	• Ions electrostatically accelerated • Hall effect (HET) • Ion • Field emission	• Plasma accelerated via interaction of current and magnetic field • Pulsed plasma (PPTs) • Magnetoplasmadynamic (MPD) • Pulsed inductive (PIT)
Power Range; 0.4–2 kW	1–50 kW	50 kW–1 MW
Specific Impulse, I_{sp}; 300–800 sec	1,000–3,000 sec	2,000–5,000 sec

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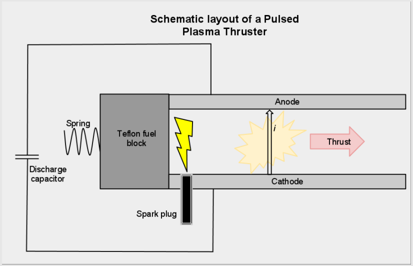
Specific Impulse, I_{sp}; 300–800 sec	1,000–3,000 sec	2,000–5,000 sec

- SRP average generation is 3.2MW.

PPT's have flown since 1964. In 2000, NASA's research PPT generated c=13,700 m/s (Isp=1,370), with thrust of 860 micro-N, requiring power of 70 W.



Electric Propulsion

Concept	Characteristics					
	Specific Impulse, (sec)	Input Power, (kW)	Thrust/ Power, (mN/kW)	Specific Mass, (kg/kW)	Propellant	Supplier
Resistojet	296	0.5	743	1.6	N_2H_4	Primex
	299	0.9	905	1	N_2H_4	Primex, TRW
Arcjet	480	0.85	135	3.5	NH_3	IRS/ITT
	502	1.8	138	3.1	N_2H_4	Primex
	>580	2.17	113	2.5	N_2H_4	Primex
	800	26*	—	—	NH_3	TRW, Primex, CTA
Pulsed Plasma Thruster (PPT)	847	< 0.03†	20.8	195	Teflon	JHU/APL
	1,200	< 0.02†	16.1	85	Teflon	Primex, TSNIIMASH, NASA
Hall Effect Thruster (HET)	1,600	1.5	55	7	Xenon	IST, Loral, Fakel
	1,638	1.4*	—	—	Xenon	TSNIIMASH, NASA
	2,042	4.5	54.3	6	Xenon	SPI, KeRC
Ion Thruster (IT)	2,585	0.5	35.6	23.6	Xenon	HAC
	2,906	0.74	37.3	22	Xenon	MELCO, Toshiba
	3,250	0.6	30	25	Xenon	MMS
	3,280	2.5	41	9.1	Xenon	HAC, NASA
	3,400	0.6	25.6	23.7	Xenon	DASA

2025-04-01

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Teflon melts at 327° C (260° C for cooking). Boils at 400° C

Staging

Previously, we assumed the rocket only consisted of payload and propellant: $m_0 = m_L + m_p$.

$$\frac{m_p}{m_0} = 1 - e^{-\frac{\Delta v}{c}} \qquad \Delta v = c \ln\left[\frac{m(t_0)}{m(t_f)}\right] = c \ln\left[\frac{m_L + m_p}{m_L}\right]$$

Which would mean the only way to increase Δv is to decrease payload or increase the size of the rocket.

However: Payload is not the only part of the rocket.

- Rocket engines and storage tanks are heavy.
- Typically, *structure* accounts for $\cong 1/7$ of the propellant weight

$$m_0 = m_L + m_s + m_p = (m_L + 1/7 m_p) + m_p$$

- While $1/7$ may not seem a lot, without staging, it limits the total Δv to

$$\Delta v = c \ln\left(\frac{m_p}{m_p/7}\right) = c \ln 7 \cong 2c \cong 6km/s \qquad (\text{assuming } m_L = 0)$$

But $\Delta v = 8$km/s is needed for low earth orbit (LEO) - not accounting for drag or gravity losses (2km/s)!

OMG: Space flight is IMPOSSIBLE!

- It's a conspiracy.

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OMG: Space flight is IMPOSSIBLE!

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This is why Hohmann was so excited about relatively small changes in c

$$\Delta v_{\max} = c \ln 7 \cong 2c \cong 6 km/s \qquad (\text{assuming } m_L = 0)$$

Structural Mass on Saturn V

Structural Mass on Saturn V

[From ESA] - Cameras mounted on the Soyuz Fregat upper stage that sent Sentinel-1A into space on 3 April 2014. It shows liftoff, the stages in the rocket's ascent and the Sentinel-1A satellite being released from the Fregat upper stage to start its life in orbit around Earth.

The 2.3 tonne satellite lifted off on a Soyuz rocket from Europe's Spaceport in Kourou, French Guiana at 21:02 GMT (23:02 CEST). The first stage separated 118 sec later, followed by the fairing (209 sec), stage 2 (287 sec) and the upper assembly (526 sec). After a 617 sec burn, the Fregat upper stage delivered Sentinel into a Sun-synchronous orbit at 693 km altitude. The satellite separated from the upper stage 23 min 24 sec after liftoff.

Structural Coefficient

Definition 2.

The ratio of structure to total mass is called the **structural coefficient**, ϵ:

$$\epsilon = \frac{m_s}{m_s + m_p}$$

In the ideal case, structural weight would be discarded as soon as it is no longer required.

- Continuous Staging

In this ideal scenario, we would have

$$\Delta v = (1-\epsilon) c \ln \left[\frac{m_0}{m_L} \right]$$

- The structure simply decreases the efficiency of the fuel!

In **Staging**, we discard structure at discrete points in time.

- Staging can never be better than $\Delta v = (1-\epsilon) c \ln \left[\frac{m_0}{m_L} \right]$.

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- Staging can never be better than $\Delta v = (1-\epsilon) c \ln\left[\frac{m_0}{m_L}\right]$.

- Prussing uses structural coefficient ϵ
- Prussing uses the term “mass ratio” to refer to the full weight of a stage over the empty weight (wet weight over dry weight).

$$Z = \frac{m_p + m_s + m_L}{m_s + m_L} = \frac{1+\lambda}{\epsilon + \lambda}$$

- Prussing uses the term “payload ratio” to indicate the ratio of payload to structural mass plus propellant mass.

$$\lambda = \frac{m_L}{m_p + m_s}$$

- I use the term “**structural mass fraction**” to indicate the mass of structure needed for every mass of fuel

$$\eta := \frac{m_s}{m_p} = \frac{\epsilon}{1-\epsilon}$$

Δv for staging

Total Δv is the sum of the Δv's from each stage:

$$\Delta v = \Delta v_1 + \Delta v_2 + \Delta v_3$$

So the Δv of each stage is

$$\Delta v_i = c \ln \frac{m_{0,i}}{m_{f,i}} = c \ln \frac{m_{0,i}}{m_{L,i} + m_{s,i}} = c \ln Z_i = c \ln \left(\frac{1 + \lambda_i}{\lambda_i + \epsilon_i} \right)$$

The total mass of each state is the payload, propellant and structural mass

$$m_{0,i} = m_{p,i} + m_{s,i} + m_{L,i}$$

The payload mass for each stage is the total mass of the following stages

$$m_{L,i} = m_{0,i+1} = m_{p,i+1} + m_{s,i+1} + m_{L,i+1}$$

When designing a multi-stage rocket: the only thing you are allowed to choose is $m_{p,i}$. These are the variables. Everything else is fixed by these choices.

$$m_{s,i} = \frac{\epsilon_i}{1 - \epsilon_i} m_{p,i} = \eta_i m_{p,i}$$

3-stage Scenario

Suppose we divide the structural and propulsive weight into three components

1. First Stage: m_{s1}, m_{p1}
2. Second Stage: m_{s2}, m_{p2}
3. Third Stage: m_L, m_{p3}

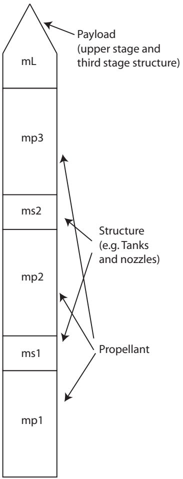
Then Δv is the combined Δv of all three stages.

$$\begin{aligned}\Delta v_T &= \Delta v_1 + \Delta v_2 + \Delta v_3 (m_L \text{ here includes } m_{s3}) \\ &= c \ln\left[\frac{m_{p1}+m_{p2}+m_{p3}+m_{s1}+m_{s2}+m_L}{m_{p2}+m_{p3}+m_{s1}+m_{s2}+m_L}\right] \\ &\quad + c\ln\left[\frac{m_{p2}+m_{p3}+m_{s2}+m_L}{m_{p3}+m_{s2}+m_L}\right] + c\ln\left[\frac{m_{p3}+m_L}{m_L}\right]\end{aligned}$$

Optimal choice of m_{p1}, m_{p2} and m_{p3} is difficult.
For a fixed total mass, m_0, we can maximize

- Payload weight
- Total Δv

A good rule of thumb is $m_{p1} = 3m_{p2} = 9m_{p3}$.



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$$\Delta v_T = \Delta v_1 + \Delta v_2 + \Delta v_3 (m_L \text{ here includes } m_{s3})$$
$$= c \ln \left[\frac{m_{p1} + m_{p2} + m_{p3} + m_{s1} + m_{s2} + m_L}{m_{p2} + m_{p3} + m_{s1} + m_{s2} + m_L} \right] + c \ln \left[\frac{m_{p2} + m_{p3} + m_{s2} + m_L}{m_{p3} + m_{s2} + m_L} \right] + c \ln \left[\frac{m_{p3} + m_L}{m_L} \right]$$

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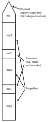

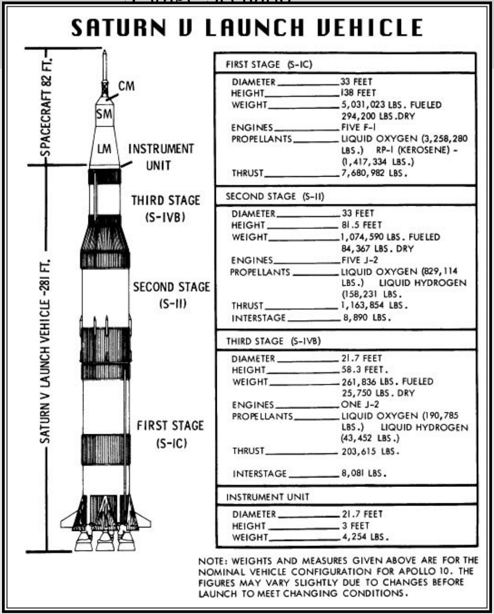

Structural coeffiecient by stage (assuming interstage is structural mass on previous stage:)

$\epsilon_1 = .0602$
$\epsilon_2 = .0868$
$\epsilon_3 = .0985$

figure from National Air and Space Museum

Comparison: Ariane IV (1988):

$\epsilon_1 = .0696$
$\epsilon_2 = .0957$
$\epsilon_3 = .01008$

mass$=500,000 - 1,000,000 lb$

Comparison: BFR (Starship+Superheavy)

$\epsilon_1 = .0651$ (est.)
$\epsilon_2 = .0909$

mass$=11,000,000 lb$

1,2 and 3stage Scenarios (Fixed Total Mass)

TABLE 6.3 ONE-, TWO-, AND THREE-STAGE ROCKETS STAGES (EQUAL ε AND λ) $c = 3048$ m/sec ($I_{sp} = 311$ sec)

	1 stage	2 stage	3 stage	
			specified m_L	specified Δv
m_{01}	15,000	15,000	15,000	15,000
m_{02}	–	3,873	6,082	4,926
m_{03}	–	–	2,466	1,618
m_{s1}	2,000	1,590	1,274	1,393
m_{s2}	–	410	517	457
m_{s3}	–	–	209	150
m_{p1}	12,000	9,537	7,644	8,681
m_{p2}	–	2,463	3,099	2,851
m_{p3}	–	–	1,257	936
m_L	1,000	1,000	1,000	531
ε	0.143	0.143	0.143	0.138
λ	0.0714	0.348	0.682	0.489
Δv (m/sec)	4,906	6,157	6,515	7,905
m_{sTOT}	2,000	2,000	2,000	2,000
m_{pTOT}	12,000	12,000	12,000	12,469
Z	5	2.75	2.039	2.374

Staging on Minuteman ICBM

Summary

This Lecture you have learned:

Introduction to Rocketry

- Mass Consumption
- Specific Impulse and Rocket Types
- Δv limitations
- Staging