Lecture 12: Orbital Perturbations
In this Lecture, you will learn:

Perturbation Basics
- The Satellite-Normal Coordinate System
- Equations for \( \dot{a}, \dot{i}, \dot{\Omega}, \dot{\omega}, \dot{e} \)

Drag Perturbations
- Models of the atmosphere.
- Orbit Decay
- \( \Delta v \) budgeting.
- Effect on eccentricity.
Introduction to Perturbations

So far, we have only discussed idealized orbits.

- Solutions to the 2-body problem.
- All orbital elements are fixed (except $f$).

In reality, there are many other forces at work:

- Drag
- Non-spherical Earth
- Lunar Gravity
- Solar Radiation
- Tidal Effects
Perturbations can be good or bad.

Perturbations allow us to break free of the $\Delta v$ budget.

There is not much flexibility in the restricted two-body problem. All maneuvering is accomplished using $\Delta v$ budget (Gravity assist being an exception)

Perturbations allow us to identify new forces which, if used correctly, can reduce our dependency on $\Delta v$ budget.
Generalized Perturbation Analysis
Satellite-Normal Coordinate System

By definition, perturbations don’t point to the center of mass

• Where do they point?
• Need a new coordinate system.

\[ \vec{F} = N\hat{e}_N + R\hat{e}_R + T\hat{e}_T \]

Satellite-Normal CS (R-T-N):

• \( \hat{e}_R \) points along the earth → satellite vector.
• \( \hat{e}_N \) points in the direction of \( \vec{h} \)
• \( \hat{e}_T \) is defined by the RHR
  \[ \hat{e}_T \cdot v > 0. \]
Generalized Perturbation Analysis

Now suppose we have an expression for the disturbing force:

\[ \vec{F} = R\hat{e}_R + T\hat{e}_T + N\hat{e}_N \]

How does this affect \( \dot{a}, \dot{i}, \dot{\Omega}, \dot{\omega}, \dot{\epsilon} \)?

Most elements depend on \( h \) and \( E \):

\[
\begin{align*}
    a &= -\frac{\mu}{2E} \\
    e &= \sqrt{1 + \frac{2Eh^2}{\mu^2}} \\
    \cos i &= \frac{h_z}{h} \\
    \tan \Omega &= \frac{h_x}{h_y} \\
\end{align*}
\]
Generalized Perturbation Analysis

Now suppose we have an expression for the disturbing force:

$$\vec{F} = R\hat{e}_R + T\hat{e}_T + N\hat{e}_N$$

How does this affect $\dot{a}$, $\dot{i}$, $\dot{\Omega}$, $\dot{\omega}$, $\dot{e}$?

Most elements depend on $\vec{h}$ and $E$:

$$a = -\mu^2 E e$$

$$e = \sqrt{1 + \frac{2}{\mu^2} Eh^2}$$

$$\cos i = \frac{h_z}{h}$$

$$\tan \Omega = \frac{h_x - h_y}{h_z}$$

Here we see the direct relationship between physical parameters $h, E$ and orbital parameters $a, e$.

In the presence of perturbations, angular momentum and energy of the satellite are not conserved.

Hence, in the presence of perturbations, the orbit is no longer truly elliptic. Hence the orbital elements are not perfect parameters of motion. However, deviations from the ellipse occur over long time-horizons and so we assume a quasi-stationary elliptic motion and include adjustments to the ellipse in the form of $\dot{a}, \dot{i}, \dot{\Omega}, \dot{\omega}, \dot{e}$. Also, we don’t have anything better.
Energy and Momentum Perturbation

We have the orbital elements in terms of $\vec{h}$ and $E$.

1. Find expressions for $\dot{\vec{h}}$ and $\dot{E}$.
2. Translate into expressions for $\dot{a}$, $e$, etc.

**Example 1:** Semimajor axis.

$$a = -\frac{\mu}{2E}$$

Chain Rule:

$$\dot{a} = \frac{da}{dE} \frac{dE}{dt} = \frac{\mu}{2E^2} \dot{E}$$

**Example 2:** Eccentricity.

$$e = \sqrt{1 + \frac{2E\vec{h}^2}{\mu^2}}$$

Chain Rule:

$$\dot{e} = \frac{de}{dh} \frac{dh}{dt} + \frac{de}{dE} \frac{dE}{dt} = \frac{1}{2e} (e^2 - 1) \left[ \frac{\dot{h}}{h} - \frac{\dot{E}}{E} \right]$$
Energy and Momentum Perturbation

So now the key is to find expressions for $\dot{h}$ and $\dot{E}$. Let $\vec{F}$ be the disturbing force per unit mass (watch those units!) in RTN coordinates:

$$\vec{F} = \begin{bmatrix} R \\ T \\ N \end{bmatrix}$$

**Energy:** Energy is Force times distance.

$$dE = \vec{F} \cdot d\vec{r}$$

So in RTN coordinates,

$$\dot{E} = \vec{F} \cdot \vec{v} = \vec{F} \cdot \left( \dot{r}\hat{e}_R + r\dot{\theta}\hat{e}_T \right) = \dot{r}R + r\dot{\theta}T$$

**Momentum:** Newton’s Second Law:

$$\dot{\vec{h}} = \vec{r} \times \vec{F} = rT\hat{e}_N - rN\hat{e}_T$$

With magnitude

$$\dot{h} = \frac{\vec{h} \cdot \vec{v}}{\vec{h}} = \frac{(h\hat{e}_N) \cdot (rT\hat{e}_N - rN\hat{e}_T)}{h^2} = \frac{rT}{h \hat{e}_N}$$
Energy and Momentum Perturbation

So now the key is to find expressions for $\dot{h}$ and $\dot{E}$. Let $\vec{F}$ be the disturbing force per unit mass (watch those units!) in RTN coordinates:

$$\vec{F} = \begin{bmatrix} F^R \\ F^T \end{bmatrix}$$

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So in RTN coordinates,

$$\dot{E} = \vec{F} \cdot \vec{v} = \begin{bmatrix} F^R \\ F^T \end{bmatrix} \begin{bmatrix} \dot{r}^R \\ \dot{r}^T \end{bmatrix} = \dot{r}^R \hat{e}_R + r \dot{\theta} \hat{e}_T$$

**Momentum:** Newton’s Second Law.

$$\dot{\vec{h}} = \vec{r} \times \vec{F} = \begin{bmatrix} F^T \\ -F^R \end{bmatrix} = r \dot{\theta} \hat{e}_N$$

With magnitude

$$\dot{h} = \vec{h} \cdot \dot{\vec{h}} = h \dot{e}_N = \frac{h^2}{r}$$

Energy is NOT conserved. Some disturbances can sap energy (e.g. drag).

Some can increase energy (e.g. solar wind)

We have assumed quasi-elliptic motion, so...

Recall $\vec{v} = \dot{r} \hat{e}_R + r \dot{\theta} \hat{e}_T$ is the velocity in RTN - recall Lecture 2!

Recall $\vec{r}$ is always in the orbital plane! So $\hat{e}_N \cdot \vec{r} = 0$.

Also recall $\vec{h} = h \hat{e}_N$. 

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Energy and Momentum Perturbation

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$$\vec{F} = \begin{bmatrix} F^R \\ F^T \end{bmatrix}$$

**Energy:** Energy is Force times distance.

$$dE = \vec{F} \cdot d\vec{r}$$

So in RTN coordinates,

$$\dot{E} = \vec{F} \cdot \vec{v} = \begin{bmatrix} F^R \\ F^T \end{bmatrix} \begin{bmatrix} \dot{r}^R \\ \dot{r}^T \end{bmatrix} = \dot{r}^R \hat{e}_R + r \dot{\theta} \hat{e}_T$$

**Momentum:** Newton’s Second Law.

$$\dot{\vec{h}} = \vec{r} \times \vec{F} = \begin{bmatrix} F^T \\ -F^R \end{bmatrix} = r \dot{\theta} \hat{e}_N$$

With magnitude

$$\dot{h} = \vec{h} \cdot \dot{\vec{h}} = h \dot{e}_N = \frac{h^2}{r}$$
Semi-Major Axis Perturbation

Using \( r = \frac{h^2/\mu}{1 + e \cos f} \) and the approximation \( \dot{\theta} = \frac{d}{dt}(\omega + f) \approx \dot{f} = \frac{h}{r^2} \), we combine

\[
\dot{a} = \frac{\mu}{2E^2} \dot{E}
\]

with

\[
\dot{E} = \dot{r}R + r\dot{\theta}T
\]

where \( E = -\frac{\mu}{2a} \) to get:

**Semi-major Axis**

\[
\dot{a} = 2\frac{a^2}{\mu} \left[ R \frac{ae \sin f}{h} + T \frac{h}{r} \right]
\]

or, in terms of \( a, e, \) and \( f \),

\[
\dot{a} = 2\sqrt{\frac{a^3}{\mu(1-e^2)}} [eR \sin f + T(1 + e \cos f)]
\]
Recall by definition \( h = r \cdot v \perp = r \cdot (r \dot{f}) = r^2 \dot{f} \).

Since \( r = \frac{h^2 / \mu}{1 + e \cos f} \), we have using the chain rule

\[
\dot{r} = \frac{r^2 e \sin f \dot{f}}{h^2 / \mu} = \frac{\mu e \sin f}{h}
\]
Eccentricity Perturbation

Using $r = \frac{h^2/\mu}{1 + e \cos f}$ and the approximation $\dot{\theta} = \frac{d}{dt}(\omega + f) \approx \dot{f} = \frac{h}{r^2}$, we combine

$$\dot{e} = \frac{1}{2e} (e^2 - 1) \left[ \frac{\dot{h}}{h} - \frac{\dot{E}}{E} \right]$$

with

$$\dot{E} = \dot{r} R + r \dot{\theta} T \quad \text{and} \quad \dot{h} = \dot{\theta} T$$

where $E = -\frac{\mu}{2a}$ to get

**Eccentricity:**

$$\dot{e} = \sqrt{\frac{a(1 - e^2)}{\mu}} \left[ R \sin f + T (\cos f + \cos E_{ecc}) \right]$$

where $E_{ecc}$ is eccentric anomaly,

$$\tan \left( \frac{E_{ecc}}{2} \right) = \sqrt{\frac{1 - e}{1 + e}} \tan \left( \frac{f}{2} \right)$$
Eccentricity Perturbation

Using \( r = \frac{\dot{r}}{1 - e \cos f + \frac{h}{\mu}} \) and the approximation \( \dot{r} = \dot{f} = \dot{f} \), we combine

\[
\dot{e} = \frac{1}{\mu} \left( 1 - e \right) \left( \frac{h}{\mu} \right)
\]

with

\[
\dot{E} = \dot{r} + \dot{f}
\]

where \( E = \omega + f \). Letting \( h = \mu f \), we get

\[
E_{ecc} = \frac{\sqrt{1 - e^2}}{e} \tan f
\]

where \( E_{ecc} \) is eccentric anomaly.

An alternate form of the equation:

\[
r = a \left( 1 - e \cos E_{ecc} \right)
\]
Energy and Momentum Perturbation

Inclination and RAAN

**Inclination:** From

\[ \cos i = \left( \frac{h_z}{h} \right) \]

we have from the chain rule

\[ \frac{d}{dt}i = \frac{1}{\sin i} \left( \frac{h_x}{h} \dot{h}_z - \frac{h_z}{h} \dot{h}_x \right) \]

from which we can get

\[ \frac{d}{dt}i = \sqrt{\frac{a(1 - e^2)}{\mu} \cos(\omega + f)} \]

Although complicated, we can also find \( \dot{\omega} \).

\[ \dot{\omega} = -\dot{\Omega} \cos i + \sqrt{\frac{a(1 - e^2)}{e^2 \mu}} \left( -R \cos f + T \frac{(2 + e \cos f) \sin f}{1 + e \cos f} \right) \]

**RAAN:** From

\[ \tan \Omega = \frac{h_x}{-h_y} \]

we have from the chain rule

\[ \dot{\Omega} = \cos^2 \Omega \left( \frac{h_x h_y - h_z^2}{h_x^2 h_y} \right) \]

from which we can get

\[ \dot{\Omega} = \sqrt{\frac{a(1 - e^2)}{\mu} \sin(\omega + f)} \sin i \left( 1 + e \cos f \right) \]
Energy and Momentum Perturbation

Inclination:
From \( \cos i = \frac{h_z}{h} \) we have from the chain rule
\[
\frac{d}{dt}i = \sin i \dot{h}_z - \dot{h}_h h_z h^2
\]
from which we can get
\[
\frac{d}{dt}i = \sqrt{a \left( 1 - e^2 \right)} \frac{\mu N}{\cos (\omega + f)} \left( 1 + e \cos f \right)
\]

RAAN:
From \( \tan \Omega = \frac{h_x - h_y}{h} \) we have from the chain rule
\[
\dot{\Omega} = \cos^2 \Omega \dot{h}_x \dot{h}_y - \dot{h}_x h_y h^2
\]
from which we can get
\[
\dot{\Omega} = \sqrt{a \left( 1 - e^2 \right)} \frac{\mu N}{\sin (\omega + f)} \sin i \left( 1 + e \cos f \right)
\]

Although complicated, we can also find \( \dot{\omega} \).
\[
\dot{\omega} = -\dot{\Omega} \cos i + \sqrt{a \left( 1 - e^2 \right)} e \frac{\mu}{\mu} \left( -R \cos f + T \left( 1 + e \cos f \right) \sin f \left( 1 + e \cos f \right) \right)
\]

Use Rotation matrices to convert:
\[
\dot{\hat{h}} = \vec{r} \times \vec{F} = \dot{r} \times \vec{F}
\]
\[
= rT\hat{e}_N - rN\hat{e}_T = \begin{bmatrix} 0 \\ -rN \\ rT \end{bmatrix}_{RNT}
\]
\[
= \begin{bmatrix} \dot{h}_x \\ \dot{h}_y \\ rT \cos i - rN \cos \theta \sin i \end{bmatrix}_{ECI}
\]

Where in the last step, we used the rotation matrix \( R_{RTN \rightarrow ECI} = R_3(\Omega)R_1(i)R_3(\theta) \) from Lecture 7. However, the expression for \( \dot{h}_x, \dot{h}_y \) is too complicated for these slides.
**Problem:** Suppose a satellite of 100kg in circular polar orbit of 42,164km experiences a continuous solar pressure of .1 Newton in $\hat{e}_N$ direction. How do the orbital elements vary with time?

**Solution:** The Force per unit mass is $N = F/m = .001m/s^2 = 1E-6 km/s^2$.

Since $T = R - e = 0$, and $f \approx E_{ecc} \approx M = nt$

\[\dot{a} = 2\sqrt{\frac{a^3}{\mu(1-e^2)}} [eR \sin f + T(1 + e \cos E)] = 0\]

\[\dot{e} = \sqrt{\frac{a(1-e^2)}{\mu}} [R \sin f + T(\cos f + \cos E_{ecc})] = 0\]

For inclination, we have

\[\frac{d}{dt}i = N \sqrt{\frac{a(1-e^2) \cos(\omega + f)}{\mu}} \cos nt = N \sqrt{\frac{a}{\mu}} \cos nt\]
Levitated Orbit Example

The formula for inclination integrates out to

\[ \Delta i(t) = N \sqrt{\frac{a}{\mu n}} \sin nt = 0.00446 \sin nt \text{ radians} \]

Similarly, since \( i \approx 90^\circ \)

\[ \dot{\Omega} = N \sqrt{\frac{a(1 - e^2)}{\mu}} \sin(\omega + f) \frac{\sin i(1 + e \cos f)}{\sin i} = N \frac{a_i}{\mu} \sin nt \]

We have

\[ \Delta \Omega(t) = -N \sqrt{\frac{a}{\mu n}} \cos nt = -0.00446 \cos nt \text{ radians} \]

The effect is a "Displaced" orbit. The size of the displacement is \( 0.0045 \text{rad} \times 42164 \text{ km} = 188 \text{km} \). See "Light Levitated Geostationary Cylindrical Orbits are Feasible" by S. Baig and C. R. McInnes.
Levitated Orbit Example

The formula for inclination integrates out to
\[ \Delta i(t) = N \sqrt{\frac{a}{\mu}} \sin nt = 0.00446 \sin ntradians. \]

Similarly, since \( i \sim 90^\circ \)
\[ \dot{\Omega} = N \sqrt{\frac{a}{\mu}} (1 - e^2) \sin(\omega + f) \sin i (1 + e \cos f) = N \sqrt{\frac{a}{\mu}} \sin nt. \]

We have
\[ \Delta \Omega(t) = -N \sqrt{\frac{a}{\mu}} \cos nt = -0.00446 \cos ntradians. \]

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- At ascending node, pulled forward \(+\hat{e}_N\) by 188km due to \(+\Delta \Omega\), no \( \Delta i \)
- At descending node, pulled forward \(+\hat{e}_N\) by 188km due to \(-\Delta \Omega\), no \( \Delta i \)
- At north pole, pulled forward \(+\hat{e}_N\) by 188km due to \(+\Delta i\), no \( \Delta \Omega \)
- At south pole, pulled forward \(+\hat{e}_N\) by 188km due to \(-\Delta i\), no \( \Delta \Omega \)
Periodic and Secular Variation

The preceding example illustrated the effect of periodic variation.

There are three types of disturbances:

- **Short Periodic** - Cycles every orbital period.
- **Long Periodic** - Cycles last longer than one orbital period.
- **Secular** - Does not cycle. Disturbances mount over time.

Secular Disturbances must be corrected.

*Example: Drift in the ECI (Earth-Centered Inertial) frame.*
Earth’s atmosphere extends into space.

The ionosphere extends well past 350km.

- ISS orbit lies between 330 and 400km.
It's called the ionosphere because all the atmospheric gasses have lost their electrons.
The Ionosphere

Figure: The Aurora Borealis Shows the Ionosphere Extending Well into Orbital Range
The Drag Perturbation

Drag force for satellites is the same as for aircraft

\[ F_D = C_D Q A = \frac{1}{2} \rho v^2 C_D A \]

By definition, drag is opposite to the velocity vector.

- Since by definition, \( \vec{v} \perp \vec{h} \), \( N = 0 \)
- For now, ignore the rotation of the earth (adds \( \Delta v = \omega_e r \approx 0.5 \text{ km/s} \)).
- For now, assume circular orbit, so \( \vec{v} = v \hat{e}_T \).

**Ballistic Coefficient:**

\[ B = \frac{m}{C_D A} \]

Then as first approximation,

\[ N = R = 0, \quad T = -\frac{1}{2} \frac{\rho}{m} C_D A v^2 = -\frac{1}{2} \frac{\rho v^2}{B} \]
The Drag Perturbation

Drag force for satellites is the same as for aircraft

\[ F_D = C_D Q A = \frac{1}{2} \rho v^2 C_D A \]

By definition, drag is opposite to the velocity vector.

• Since by definition, \( \vec{v} \perp \vec{h} \), \( N = 0 \)
• For now, ignore the rotation of the earth (adds \( \Delta v = \omega_e r \sim 5 \text{ km/s} \)).
• For now, assume circular orbit, so \( \vec{v} = v \hat{e}_T \).

Ballistic Coefficient:

\[ B = \frac{m}{C_D A} \]

Then as first approximation,

\[ N \approx 0, \quad T = -\frac{1}{2} \rho v^2 C_D A v = -\frac{1}{2} \rho v^2 B^2 \]

- \( Q \) is dynamic pressure.
- \( \vec{v} \) is in the orbital plane and \( \vec{h} \) is perpendicular to the orbital plane.
- \( A \) is the area of the spacecraft projected onto the \( \hat{e}_N - \hat{e}_R \) plane.
- \( C_D \) measures how aerodynamic the spacecraft is.
- Drag can also generate lift (\( C_L \))! A component in the \( \hat{e}_R \) direction (or even the \( \hat{e}_N \) direction)
The Drag Effect on Orbital Elements

Circular Orbits, Constant Density

First note that since $N = 0$, the orbital plane does not change

- $\dot{\Omega} = 0$
- $\frac{d}{dt} i = 0$

**Semi-Major Axis:** Since $e = 0$, the dominant effect is on $a$.

$$\dot{a} = 2\sqrt{\frac{a^3}{\mu(1-e^2)}} \left[ eR \sin f + T(1 + e \cos f) \right]$$

Integrating with respect to time (assuming constant $\rho$) yields

$$a(t) = \left( \sqrt{a(0)} - \sqrt{\frac{\mu}{B}} t \right)^2$$
The Drag Effect on Orbital Elements

- Circular Orbits, Constant Density

First note that since $N = 0$, the orbital plane does not change.

- $\dot{\Omega} = 0$.
- $d/dt i = 0$.

Semi-Major Axis:

Since $e = 0$, the dominant effect is on $a$.

$$\dot{a} = 2 \sqrt{a^3} \mu \left(1 - e^2\right) \left[eR \sin f + T(1 + e \cos f)\right],$$

$$= -\sqrt{a^3} \mu \rho \frac{C_D}{2} \vspace{1em}$$

Integrating with respect to time (assuming constant $\rho$) yields

$$a(t) = \left(\sqrt{a(0)} - \sqrt{\mu \rho B} t\right)^2,$$

$\hat{v} = \sqrt{\mu/r}$ for circular orbits.

Unfortunately, $\rho(t)$ is NOT constant.
**Example: International Space Station**

**Figure:** Orbit Decay of the International Space Station
Density Variation

The atmospheric density is not even remotely constant

**Exponential Growth:**
- Extends to $1.225 \times 10^{-3} \text{ g/cm}^3$ at sea level.
- Orbits below Kármán Line (100km) will not survive a single orbit.
  ▶ Suborbital flight.

**Solar Activity:** We have different models of the atmosphere depending on solar activity level.
- Unlike aircraft applications
- Variation mainly occurs in ionosphere
- Solar wind changes earth’s EM field
• Most density models of the atmosphere start to fail at the ionosphere.

• Kármán Line is named after Theodore van Kármán (1881–1963)

• A nominal aircraft at the Kármán Line would have to travel at orbital velocity to generate more lift than weight.

• Usually differentiates the fields of aeronautics and astronautics
All Satellites must budget $\Delta v$ (m/s/yr) to compensate for atmospheric drag. The problem with budgeting is predicting solar activity.
This data is scaled to Ballistic Coefficient.

- So if your ballistic coefficient is 10 times lower, you need 10 times the $\Delta v$!
Spacecraft Lifetime

Without stationkeeping, orbits will decay quickly.

**Definition 1.**

The **Lifetime** of a spacecraft is the time it takes to reach the 100km Kármán Line.

- The Figure shows mean value of lifetime.
- Actual values will depend on solar activity.
Spacecraft Lifetime
Solar Activity Effect

LIFETIMES FOR CIRCULAR ORBITS
(Normalized to W/CdA = 1 lb/ft**2)

Quiet atmosphere, F10.7=75

F10.7=100

F10.7=150

F10.7=200

Active, F10.7=250

Lifetime = Normalized lifetime \times \left(0.2044/C_{DA}/W\right)
Plot is normalized for a ballistic coefficient and US customary units.

- To get actual lifetime, multiply number from plot by $0.2044 \frac{W}{C_D A}$ in metric units.
Spacecraft Lifetime
Solar Activity Effect

LIFETIMES FOR CIRCULAR ORBITS
(Normalized to CdA/W = 0.2044m²/kg)

- Quiet atmosphere, F10.7=75
- F10.7=100
- F10.7=150
- F10.7=200
- Active, F10.7=250
- Lifetime = Normalized lifetime x (0.2044/CdA/W)
Solar Activity

Solar Activity varies substantially with time. $F_{10.7}$ measures normalized solar power flux at EM wavelength 10.7cm.
Solar Activity is Hard to Predict

Figure: Shatten Prediction Model with Actual Data
Drag Effects on Eccentric Orbits

Eccentric orbits are particularly prone to drag.

- Even if $a$ is large, drag at perigee is high.
- Very difficult to integrate, due to changing density
- Using Exponential Density model,
  \[
  \Delta e_{rev} = -2\pi \frac{C_D A}{m} \rho_{perigee} e^{-ae/H} \left[ I_1 + e(I_0 + I_2)/2 \right]
  \]
  
  ▶ $\rho_p$ is density at perigee. $H$ is a height constant. $I_i$ are Bessel functions
- $\Delta a$ is also complicated.
Decay of Eccentricity

Although drag occurs at perigee, apogee is lowered.
Drag Effects on Eccentric Orbits

- Altitude of perigee
- Altitude of apogee
- Eccentricity
- Period

\[ \Delta E = \text{constant} \]

- Period of 100 days
- 200 km perigee
- Circular orbit \( e = 0 \)
- Orbit of Earth

\( e = 0.12 \)
Hayabusa Re-entry
Summary

This Lecture you have learned:

Perturbation Basics
- The Satellite-Normal Coordinate System
- Equations for \( \dot{a}, \dot{i}, \dot{\Omega}, \dot{\omega}, \dot{e} \)

Drag Perturbations
- Models of the atmosphere.
- Orbit Decay
- \( \Delta v \) budgeting.
- Effect on eccentricity.

Next Lecture: Earth’s Shape and Sun-synchronous Orbits.
\[
\dot{a} = 2 \sqrt{\frac{a^3}{\mu(1-e^2)}} \left[ eR \sin f + T(1 + e \cos f) \right]
\]

\[
\dot{e} = \sqrt{\frac{a(1-e^2)}{\mu}} \left[ R \sin f + T(\cos f + \cos E_{ecc}) \right]
\]

\[
\frac{di}{dt} = \sqrt{\frac{a(1-e^2)}{\mu}} \frac{N \cos(\omega + f)}{1 + e \cos f}
\]

\[
\dot{\Omega} = \sqrt{\frac{a(1-e^2)}{\mu}} \frac{N \sin(\omega + f)}{\sin i(1 + e \cos f)}
\]

\[
\dot{\omega} = -\dot{\Omega} \cos i + \sqrt{\frac{a(1-e^2)}{e^2 \mu}} \left( -R \cos f + T \frac{(2 + e \cos f) \sin f}{1 + e \cos f} \right)
\]

Drag (circular orbit):

\[
N = R = 0, \quad T = -\frac{1}{2} B \rho v^2 = -\frac{1}{2} B \rho \frac{\mu}{a}.
\]