# MAE 509: LMIs in Optimal and Robust Control

## Syllabus

Instructor: M. Peet
email: mpeet@asu.edu
Zoom link: https://asu.zoom.us/j/2031490735
website: control.asu.edu
phone: 480-965-5599
Meeting Time: M W 4:30-5:45 in COOR 186
Office Hours: F 4:30-5:30PM (online only)
Outside Office Hours: Appointment by email or discord

**Introduction**   Please watch the videos: Intro Part 1 and Intro Part 2 for an introduction to the material and an overview of the course organization.

**Course Website**   All slides and recordings used in the class will be posted on the course website http://control.asu.edu/MAE509_frame.htm. You are encouraged to bookmark this page and notify me if any material here becomes out-of-date.

**Motivation**   The recent introduction of Linear Matrix Inequality (LMI) methods in control has dramatically expanded the types and complexity of the systems we can control. For example, consider the problems: Gain Scheduling for Missile Attitude Control (A switched system); Control of Robots over Noisy Communication Channels (a sampled-data systems); Remote Control of Spacecraft Attitude (a delayed system); Behavioural Therapy (A system with binary inputs); or self-driving vehicles (a case of decentralized control). None of these systems can be studied using classical root-locus or PID control methods. Rather, advances in these fields have been made possible through the increased power and flexibility created by the LMI (optimization-based) approach to control.

**Recorded Lectures**   Although the course will be mostly in-person, videos of most lectures are available on the course Youtube channel: Link to Youtube Channel.

**Discord**   Please join the course discord sever at Discord Invite Link for a detailed (but tentative) schedule of the course. *Important!!!*  You must use your real name on the discord channel (or some variation thereof). Anonymous or unidentifiable users will be removed from the channel.

**Course Objectives**   To be able to use LMI solvers to synthesize optimal or suboptimal controllers and estimators for multiple classes of state-space systems.

**Learning Outcomes**   Upon completion of this course, students will be able to:

1. Recognize the three parts of an optimization problem: Variables, Objective, Constraints

2. Categorize an optimization problem as convex or non-convex

3. Categorize a convex optimization problem as semidefinite programming, linear programming, second-order cone programming, etc.

4. Recognize the difference between tracking and regulator frameworks

5. Construct the 9-matrix representation of an optimal control problem

6. Construct an $H_\infty$-optimal dynamic output feedback controller using LMI solvers

7. Implement and Test a dynamic output feedback controller on a state-space system

8. Construct the LFT representation of a robust control problem

9. Construct a robust state-feedback controller using LMI solvers

**Lecture Notes**   I am in the process of writing a textbook for this course. Participants in the course will be provided with the incomplete, partially edited version of this textbook at the beginning. As I complete additional sections of the textbook, updates will be provided. These drafts are copyrighted, confidential and all students will be asked to sign non-distribution agreements.

**Grading Policies**   There will be approximately 5 homework assignments which cumulatively will be the basis for 20% of the course grade. A midterm will be given for 30% of the grade. In-class activities, quizzes, exercises, and the wikipage project will constitute 25% of the grade. The final project or final exam will be for 25% of the grade.

**Absence Policy**   Attendance at all in-person lectures is required. Exceptions include excused absences related to university sanctioned events/activities that are in accord with ACD 304–02, "Missed Classes Due to University-Sanctioned Activities" and excused absences related to religious observances/practices that are in accord with ACD 304–04, "Accommodation for Religious Practices". In general, late homework will not be graded and missing examinations cannot be retaken. Exceptions include excused absences related to university sanctioned events/activities that are in accord with ACD 304–02, "Missed Classes Due to University-Sanctioned Activities" and excused absences related to religious observances/practices that are in accord with ACD 304–04, "Accommodation for Religious Practices"

**Content**   In this course, we provide an overview of LMIs and their many applications in Modern Control Theory. In contrast to other courses in control, our subject matter will be more focused on the variety of applications for LMIs and less on theory. Nonetheless, there is a significant theoretic component and students should expect to perform rigorous mathematical proofs.

The primary text for the class is "LMIs in Control Systems: Analysis, Design and Applications" by G.-R. Duan and H.-H. Yu. This text is not organized in the same way as the course, however. The second text we will use is "Linear Matrix Inequalities in System and Control Theory" by S. Boyd. This second text is freely available online from the author and may be found at

https://web.stanford.edu/ boyd/lmibook/lmibook.pdf.

A more theoretical treatment of some of the material can be found in "A Course in Robust Control Theory: A Convex Approach" by G. Dullerud and F. Paganini. Be warned, however, that many students find this text to be difficult to follow.

I will assume that students are familiar with the basics of state-space systems or are capable to learning about them very quickly. For an accessible text on State-Space methods, I recommend "Linear State-Space Control Systems" by D. A. Lawrence. If you have limited exposure to state-space, it might be a good idea to review the first few chapters of this book.

**Schedule**   Class meets on Monday and Wednesday from 4:30-5:45 in room COOR 185. A detailed schedule will be posted on canvas and the course discord channel.

**Lecture Format**   Pre-recorded lectures will be posted on canvas. Links to these lectures can also be found on my website `http://control.asu.edu` under the classes heading - MAE 598.

**Canvas**   Please monitor canvas for important class announcements. Lecture Notes and pre-recorded lectures will be posted on Blackboard, along with all assignments and supplementary material. Grades will also be posted on canvas.

**Prerequisites**   There is no prerequisite for this class. However, it is assumed that all students have some background in controls and linear algebra. The first two modules will review standard material on optimization and state-space systems. Access to Matlab is required, including the robust control toolbox.

**Final Project**   In lieu of a final exam, this class will have a final project. This project is expected to be development of a new LMI. Part of the project will be documentation of this LMI on the course Wikibook. This project may be based on your own research or may be on an entirely new topic. The project should leverage LMI methods to solve a problem which has not already been solved. Some problem suggestions are listed at the end of this syllabus. You may work in groups of up to 2 people. However, if you work in a group of 2, I will expect double the results as of a group of 1. All project topics should be submitted to me in advance. If you do not wish to do a project, you may take a final exam instead.

**Classroom Behavior**   Students are expected to participate in in-class activities, including disputation.

**Violence**   See the Student Services Manual, SSM 104–02, "Handling Disruptive, Threatening, or Violent Individuals on Campus"

**Disabilities**   A reminder to students that when requesting accommodation for a disability, they must be registered with the Disability Resource Center (DRC) and submit appropriate documentation from the DRC

**Academic Integrity**   Collaboration on exams will result in an E for the course. Copying of homework or duplication of material found online will result in a "0" on the homework and a referral to the ASU office of academic integrity. **Reminder:** Two referrals to the office of academic integrity is grounds for expulsion from the university. If in doubt about a specific case, ask me.

**Plagiarism**   Plagiarism applies to homework, numerical code, and the final project. In all cases, it refers to the use of outside material without attribution. In the case of code, it can mean re-arrangement of lines, renaming of variables or other cosmetic changes. In the case of the final project, it includes "rephrasing" of published or unpublished material. This occasionally includes proofs. The penalty for plagiarism on exams is an E for the course. The penalty for plagiarism on homework is as defined in the paragraph on Academic Integrity.

**References**   Aside from the text, there are several excellent optional sources which may be consulted. Although not directly required for the course, students are encouraged to browse the following references.

The following is an introduction to classical control and state-space theory.

- Franklin, Powell and Enami. "Feedback Control of Dynamical Systems", Addison-Wiley, 1994.

The following are references for LMI methods in control.

- Zhou, Doyle and Glover. "Robust and Optimal Control", Prentice Hall, 1996.

- Boyd, El Ghaoui, Feron and Balakrishnan. "Linear Matrix Inequalities in Systems and Control Theory", SIAM, 1994.

The following is a thorough reference on matrix analysis.

- Horn and Johnson. "Matrix Analysis", Cambridge University Press, 1985.

The following is a clearly written text on mathematical analysis.

- Marsden and Hoffman. "Elementary Classical Analysis", Freeman 1993.

**Assignments:**

- Week 1-3 **Read:** Duan, chapter 1, 3, **Assignment:** HW 1

- Week 4-5 **Read:** Duan, chapter 4.1,4.2, 6.1, 6.2, 7.1, 7.2, 9.1.1, **Assignment:** HW 2

- Week 6-7 **Read:** Duan, chapter 8.1,8.2,8.4, **Assignment:** HW 3

- Week 8-9 **Midterm Examination**

- Week 10-11 **Read:** Duan, chapters 7.3, 8.3, 11, **Assignment:** HW 3

- Week 12-13 **Read:** Course Notes on SOS/PDEs/Switched Systems

- Week 14-15 **Final Examination**

**Outline:** The following is a rough outline of the set of lectures which will be presented.

Lecture 1 - Introduction
Lecture 3 - Introduction to Optimization
Lecture 4 - Introduction to Positive Matrices and LMIs
Lecture 5 - Our First LMI
Lecture 2 - LMIs for Stability, Controllability and Observability.
Lecture 6 - The Optimal Control Framework
Lecture 7 - An LMI for Full-State Feedback Controller Synthesis
Lecture 8 - An LMI for $H_2$-Optimal Full-State Feedback Control (LQR)
Lecture 9 - The $H_\infty$ norm
Lecture 10 - An LMI for $H_\infty$-Optimal Full-State Feedback Control
Lecture 11 - The Luenberger Observer and The Kalman Filter
Lecture 12 - An LMI for Output-Feedback Stabilization
Lecture 13 - An LMI for $H_\infty$-Optimal Output Feedback Control
Lecture 14 - Systems with Uncertainty and the Structured Singular Value
Lecture 15 - An LMI for Robust Stability Analysis
Lecture 16 - The D-K iteration
Lecture 17 - Switched Systems
Lecture 18 - An LMI for Control of Systems with Switching
Lecture 19 - Delayed Systems
Lecture 20 - An LMI for Stability of Systems with Delay
Lecture 21 - Nonlinear Systems
Lecture 22 - An LMI for Stability of Nonlinear Systems

**Examples of Proposed Project Topics:**

- Gain Scheduling for Missile Attitude Control (Switched Systems)

- Control of Robots over Noisy Communication Channels (Sampled-Data Systems)

- Spacecraft Attitude Control with delayed communication (Delay Systems)

- Social Cognitive Therapy using Discrete Inputs (Mixed-Integer Control)

- Self-Driving Vehicles (Decentralized Control)

**Incompletes** An "incomplete" may be awarded only in cases when a student, who is otherwise performing satisfactorily, cannot complete final course requirements, such as the final exam or final assignment, due to circumstances beyond the student's control (such as illness or family emergency). Such circumstances must be documented. Incompletes will be approved only within the last one or two weeks of the semester. Incompletes cannot be requested after the time of the scheduled final exam for the course. To request a grade of incomplete, the student must formally apply to the instructor using the university's "Incomplete Grade Request" form. Requests must be submitted to the student's advisor prior to the final grade due date and are subject to final approval by the program.