MAE 598: LMIs in Optimal and Robust Control

Syllabus

Instructor: M. Peet Meeting Time: MW 3:05-4:20 in ECG 236 Office Hours: MW 11:00-12:00 in ERC 253

Motivation The recent introduction of Linear Matrix Inequality (LMI) methods in control has dramatically expanded the types and complexity of the systems we can control. For example, consider problems: Gain Scheduling for Missile Attitude Control (A switched system); Control of Robots over Noisy Communication Channels (a sampled-data systems); Remote Control of Spacecraft Attitude (a delayed system); Behavioural Therapy (A system with binary inputs); or self-driving vehicles (a case of decentralized control). None of these systems can be studied using classical root-locus or PID control methods. Rather, advances in these fields have been made possible through the increased power and flexibility created by the LMI (optimization-based) approach to control.

Goal To be able to use LMI solvers to synthesize optimal or suboptimal controllers and estimators for multiple classes of state-space systems.

Content In this course, we provide an overview of LMIs and their many applications in Modern Control Theory. In contrast to other courses in control, our subject matter will be more focused on the variety of applications for LMIs and less on theory. Nonetheless, there is a significant theoretic component and students should expect to perform rigorous mathematical proofs.

The primary text for the class is "LMIs in Control Systems: Analysis, Design and Applications" by G.-R. Duan and H.-H. Yu. This text is not organized in the same way as the course, however. The second text we will use is "Linear Matrix Inequalities in System and Control Theory" by S. Boyd. This second text is freely available online from the author and may be found at

https://web.stanford.edu/ boyd/lmibook/lmibook.pdf.

A more theoretical treatment of some of the material can be found in "A Course in Robust Control Theory: A Convex Approach" by G. Dullerud and F. Paganini. Be warned, however, that many students find this text to be extremely difficult to follow.

I will assume that students are familiar with the basics of state-space systems or are capable to learning about them very quickly. For an accessible text on State-Space methods, I recommend "Linear State-Space Control Systems" by D. A. Lawrence. If you have limited exposure to state-space, it might be a good idea to review the first few chapters of this book.

Schedule Class meets on Monday and Wednesday from 3:05-4:20 in room ECG 236. Assignments will be given approximately bi-weekly. There will be a midterm and a final project.

Lecture Format Lectures will utilize a not-yet created set of LaTeX slides. Some of these slides will borrow from MAE 507, the slides for which are found online at my website http://control.asu.edu under the classes heading - MAE 507. As a new set of slides is developed, these will be posted on Blackboard and ultimately on my website.

Blackboard Lecture Notes will be posted on Blackboard, along with all assignments and supplementary material. Grades will also be posted on blackboard.

Prerequisites There is no prerequisite for this class. However, it is assumed that all students have some background in controls and linear algebra. Access to Matlab is required, including the robust control toolbox - available in the ASU computing laboratories.

Evaluation Homework will be the basis for 30% of the grade. Problem sets will be given on a bi-weekly basis. Late homework will be graded for 75% credit. A midterm will be given for 30% of the grade. The final project will be for 40% of the grade.

Final Project In lieu of a final exam, this class will have a final project. This project may be based on your own research or may be on an entirely new topic. The project should leverage LMI methods to solve a problem which has not already been solved. Some problem suggestions are listed at the end of this syllabus. You may work in groups of up to 2 people. However, if you work in a group of 2, I will expect double the results as of a group of 1. All project topics should be submitted to me in advance. If you do not wish to do a project, you must inform me in advance and a suitable alternative found.

Honest Policy Collaboration on exams will result in an E for the course. Copying of homework or duplication of material found online will result in a "0" on the homework and a referral to the ASU office of academic integrity. **Reminder:** Two referrals to the office of academic integrity is grounds for expulsion from the university. If in doubt about a specific case, ask me.

References Aside from the text, there are several excellent sources which may need to be consulted. Although not directly required for the course, students are encouraged to browse the following references.

The following is an introduction to classical control and state-space theory.

• Franklin, Powell and Enami. "Feedback Control of Dynamical Systems", Addison-Wiley, 1994.

The following are references for LMI methods in control.

- Zhou, Doyle and Glover. "Robust and Optimal Control", Prentice Hall, 1996.
- Boyd, El Ghaoui, Feron and Balakrishnan. "Linear Matrix Inequalities in Systems and Control Theory", SIAM, 1994.

The following is a thorough reference on matrix analysis.

• Horn and Johnson. "Matrix Analysis", Cambridge University Press, 1985.

The following is a clearly written text on mathematical analysis.

• Marsden and Hoffman. "Elementary Classical Analysis", Freeman 1993.

Outline: The following is a rough outline of the set of lectures which will be presented.

- Lecture 1 Introduction
- Lecture 3 Introduction to Optimization
- Lecture 4 Introduction to Positive Matrices and LMIs
- Lecture 5 Our First LMI
- Lecture 2 LMIs for Stability, Controllability and Observability.
- Lecture 6 The Optimal Control Framework
- Lecture 7 An LMI for Full-State Feedback Controller Synthesis
- Lecture 8 An LMI for H_2 -Optimal Full-State Feedback Control (LQR)
- Lecture 9 The H_{∞} norm
- Lecture 10 An LMI for $H_\infty\text{-}\mathrm{Optimal}$ Full-State Feedback Control
- Lecture 11 The Luenberger Observer and The Kalman Filter
- Lecture 12 An LMI for Output-Feedback Stabilization
- Lecture 13 An LMI for $H_\infty\text{-}\mathrm{Optimal}$ Output Feedback Control
- Lecture 14 Systems with Uncertainty and the Structured Singular Value
- Lecture 15 An LMI for Robust Stability Analysis
- Lecture 16 The D-K iteration
- Lecture 17 Switched Systems
- Lecture 18 An LMI for Control of Systems with Switching
- Lecture 19 Delayed Systems
- Lecture 20 An LMI for Stability of Systems with Delay
- Lecture 21 Nonlinear Systems

Lecture 22 - An LMI for Stability of Nonlinear Systems

Examples of Proposed Project Topics:

- Gain Scheduling for Missile Attitude Control (Switched Systems)
- Control of Robots over Noisy Communication Channels (Sampled-Data Systems)
- Spacecraft Attitude Control with delayed communication (Delay Systems)
- Social Cognitive Therapy using Discrete Inputs (Mixed-Integer Control)
- Self-Driving Vehicles (Decentralized Control)