# Spacecraft and Aircraft Dynamics

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Lecture 1: Introduction

# MMAE441: Spacecraft and Aircraft Dynamics

Syllabus

**Instructor:** Matthew Peet
**Office:** E1 - 252B
**Teaching Assistant:** TBD
**Schedule:** MW 1:50-3:05
**Grades:** Approximately:

- Homework (30%)
- Aircraft Exam (35%)
- Spacecraft Exam (35%)

**Prereqs:**
Matrix Analysis:

- eigenvalues and eigenvectors

Dynamical Systems:

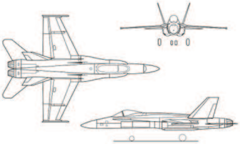
- Differential Equations
- Eigenvalues, eigenvectors and the characteristic equation
- State-Space

**Texts:**
"Orbital Mechanics", J. Prussing and B. Conway
"Flight Stability and Automatic Control", R. Nelson

- Finding Equations of Motion

$$\ddot{x}(t) = Ax(t) + B\dot{x}(t)$$

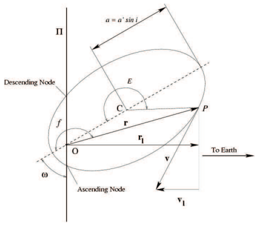
  - Identify Frames of Reference (Body-fixed, etc.)
  - Determine Coefficients (effects of wings, tail, etc.)
  - Combine effects to get EOM
- Determine Stability of Motion
  - Find natural modes (phugoid mode, etc.)
  - Relate to physical motion.
  - Determine stability.



# Introduction to Spacecraft Dynamics

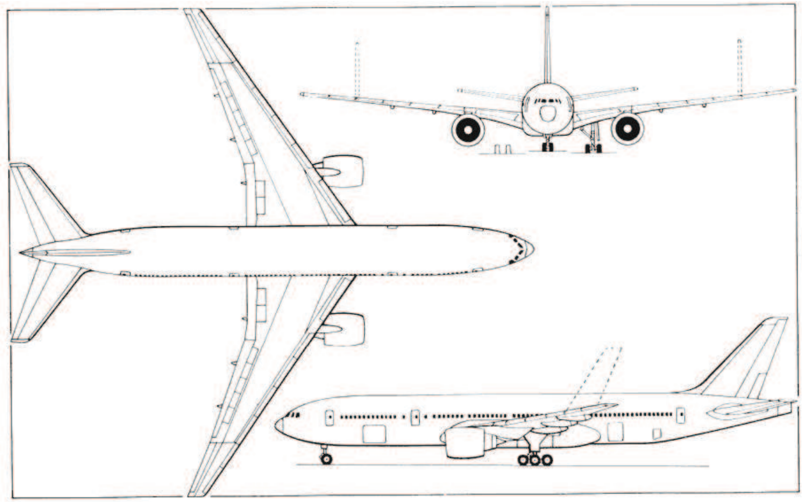
## Overview of Course Objectives

- Determining Orbital Elements
  - Know Kepler's Laws of motion, Frames of Reference (ECI, ECEF, etc.)
  - Given position and velocity, determine orbital elements.
  - Given orbital elements and time, determine position + velocity.
- Plan Earth-Orbit Transfers
  - Identify Required Orbit.
  - Find Optimal Transfer.
  - Determine Thrust and Timing.
- Plan Interplanetary Transfers
  - Design Gravity-Assist Maneuvers.
  - Use Patched-Conics.



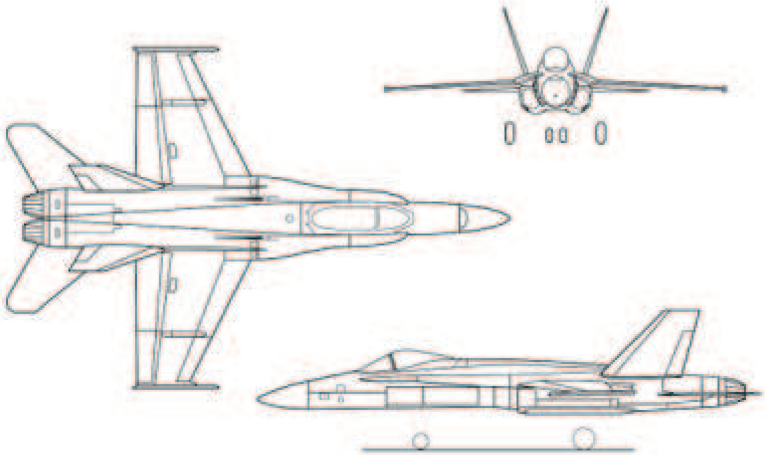
# Aircraft Dynamics

## Slideshow: Boeing 777



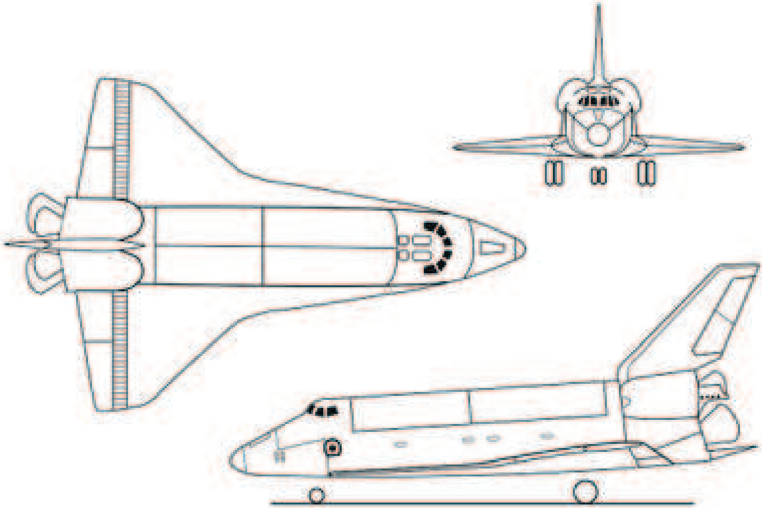
# Aircraft Dynamics

Slideshow: F/A-18



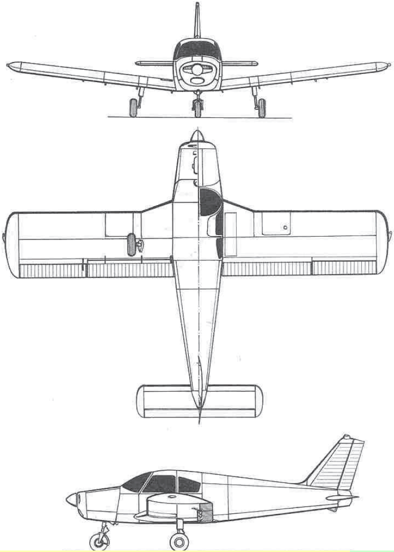
# Aircraft Dynamics

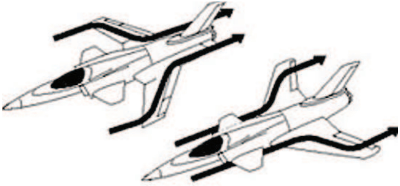
## Slideshow: NASA Space Shuttle



# Aircraft Dynamics

## Slideshow: Piper Cherokee





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# Review: Equations of Motion

## Differential Equations

The motion of dynamical systems can usually be specified using ordinary differential equations. e.g.

$$\frac{dx}{dt}(t) = f(x(t))$$

Where

- This is a first-order differential equation
- $x$ is the quantity of interest.
  - position, heading, velocity, etc.
- $f$ is a possibly nonlinear function.

**Note:** Usually, the equation is higher order or there are multiple quantities of interest.

# Review: Equations of Motion

## Linear Equations

For Aircraft Dynamics, our equations of motion will be linear. e.g.

$$\dot{x} = ax(t)$$

where

- $a$ is a constant scalar.
- in this case $f(x) = ax$.

Linear equations are preferable because

- The motion of linear systems is much easier to visualize.
- Stability of linear systems is easy to determine
  - $\dot{x} = ax$ is stable if $a < 0$ and unstable if $a \geq 0$.

# Review: Equations of Motion

## Higher Orders or Multiple Variables

Most often, the dynamics will be either

**Be coupled with another variable:**

$$\dot{x} = ax + by$$
$$\dot{y} = cx + dy$$

where

- The motion of $x$ affects the motion of $y$ and vice-versa.

**Be higher order:**

$$\ddot{x} = a\dot{x} + bx$$

where

- Commonly obtained from Newton's Third law.

$$F = ma$$

or, in other words

$$\ddot{x} = F/m.$$

# Review: Equations of Motion

## Higher Order Dynamics

When we have higher order dynamics,

$$\ddot{x} = a\dot{x} + bx$$

We often want first order dynamics if

- There are multiple variables.
- We need state-space.

**Procedure:**

- Define a new variable for every Higher Order Term (HOT) except for the the highest.
  - e.g. $\ddot{x} = y$ and $\dot{x} = z$.
- Add a new first order differential equation for each variable.
  - e.g. $\dot{x} = z$ and $\dot{z} = y$

Finally we have for our example

$$\dot{x} = y$$
$$\dot{y} = ay + bx$$

# Review: Equations of Motion

## State-Space

State-Space is a way of writing first order differential equation using matrices. We write

$$\dot{\vec{x}} = A\vec{x}$$

where $\vec{x}$ is a vector and $A \in \mathbb{R}^{n\times n}$ is a square matrix.

**Example:**

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Is equivalent to writing the three differential equations

$$\dot{x}_1 = -x_1 + x_3 \tag{1}$$

$$\dot{x}_2 = 2x_1 \tag{2}$$

$$\dot{x}_3 = -x_2 + x_3 \tag{3}$$

Writing equations in state-space has many advantages

# Review: Equations of Motion

## Multiple Variables and State-Space

Consider the system

$$\dot{x} = ax + by$$
$$\dot{y} = cx + dy$$

When we have multiple coupled equations, the best option is: **Convert to State-Space:**

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which is easily expressed as

$$\dot{\mathbf{x}} = A\mathbf{x}$$

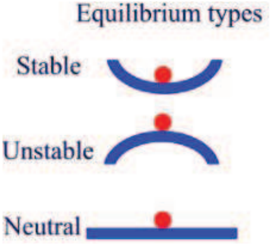
where

- $\mathbf{x}$ is a vector.
- $A$ is a matrix.

The equation describes the motion of the vector.

**Roughly Speaking:** A system of differential equations is Stable if

- small inputs produce small outputs (**Bounded-Input Bounded-Output**)
- Disturbances tend to decay (**Asymptotic Stability**)




For aircraft, we will also define **Static Stability** and **Dynamic Stability**. However, the terms *Static* and *Dynamic* refer to which equations of motion we use, and not properties of the motion itself.

# Review: Equations of Motion

## Characteristic Equation for Scalars

- Both higher-order and state-space systems have a Characteristic Equation.

The Characteristic Equation is found by using the Laplace Transform.

- $x(t) \to x(s)$
- $\dot{x}(t) \to sx(s)$
- $\ddot{x}(t) \to s^2x(s)$
- $\dddot{x}(t) \to s^3x(s)$
- $\cdots$

Thus for a scalar equation,

$$\dddot{x}(t) = a\ddot{x}(t) + b\dot{x}(t) + cx(t)$$

becomes

$$(s^3 - as^2 - bs - c)x(s) = 0$$

Therefore the characteristic equation is $s^3 - as^2 - bs - c = 0$

The roots of the characteristic equation determine the motion of the differential equation.
The roots will be **Complex**, and so will have form

$$s_0 = a + b\imath$$

where $\imath = \sqrt{-1}$.

**Stability:**

| | |
|---|---|
| Stable | Roots all have negative real part |
| Unstable | At least one root has positive real part |

**Oscillation:**

| | |
|---|---|
| Not Oscillate | All roots are real |
| Oscillate | At least one root has nonzero imaginary part |

# Review: Equations of Motion

## Characteristic Equation and Stability: Example

### A Useful Tool:

Remember the Quadratic Formula: $as^2 + bs + c$ has roots

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Example:** $s^2 + s + 1 = 0$ has roots

$$s_1 = -\frac{1}{2} + \sqrt{3}\imath$$

and

$$s_1 = -\frac{1}{2} - \sqrt{3}\imath.$$

Hence

- $\mathrm{Re}(s_1) = \mathrm{Re}(s_2) = -1$, so system is stable.
- $\mathrm{Im}(s_1) \neq 0$, $\mathrm{Im}(s_2) \neq 0$, so system is oscillatory.

$$\dot{x}(t) = Ax(t)$$

For state space, we also apply the Laplace transform to get.

$$(sI - A)x(s) = 0$$

Because $sI - A$ is matrix-valued, the characteristic equation is actually

$$\det(sI - A) = 0$$

Recall how to compute the determinant:

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

and

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \cdot \det \begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \cdot \det \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \cdot \det \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

# Review: Equations of Motion

## Example

A Spring-mass system:

$$F = -kx - cv$$

Use $k = 50$, $c = 15$, and mass $m = 1$. Then $v = \dot{x}$ and $F = ma = \ddot{x}$ so

$$\ddot{x}(t) = -15\dot{x}(t) - 50x(t)$$

which has characteristic equation

$$s^2 + 15s + 50 = 0$$

which has roots at $s = -5$ and $s = -10$. Hence the system is stable, non oscillatory.

# Review: Equations of Motion

## Example, continued

Putting this example in state-space, we use $\ddot{x} = \dot{v}$ and $\dot{x} = v$ to get

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -50 & -15 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}$$

For the characteristic equation

$$\det(sI - A) = \det \begin{bmatrix} s & -1 \\ 50 & s+15 \end{bmatrix} = s(s+15) + 50 = s^2 + 15s + 50$$

So the characteristic equation is $s^2 + 15s + 50$, which, of course, has roots at $-5, -10$

# Review: Equations of Motion

## Characteristic Equation and Eigenvalues

In state-space, there is an easier way to find the roots of the characteristic equation: Eigenvalues.

- Any $n \times n$ matrix, $A$, has $n$ eigenvalues. Call them $\lambda_i$
- Associated with each eigenvalue, $\lambda_i$, there is an eigenvector, $x_i$.
- Eigenvalues and eigenvectors of $A$ satisfy

$$Ax_i = \lambda_i x_i$$

- The $x_i$ are the "natural" directions of $A$.
- The $\lambda_i$ are the action of $A$ on $x_i$.

# Review: Equations of Motion

## Characteristic Equation and Eigenvalues continued

For the dynamical system

$$\dot{x}(t) = Ax(t),$$

- The eigenvalues of $A$ are the roots of the characteristic equation $\det(sI - A)$.
- The properties of the eigenvalue $\lambda_i$ describe the motion in the direction $x_i$.

Eigenvalues and Eigenvector are easily computed using the Matlab command:

```
[V L]=eigs(M)
```

where

- The columns of $L$ are the eigenvectors of $M$.
- The diagonals of $V$ are the eigenvalues of $M$ listed in the same order as the eigenvectors were.

# Review: Equations of Motion

## Characteristic Equation and Eigenvalues, Example

**Example:** Take the randomly generated system

$$\dot{x} = \begin{bmatrix} 3 & 2 & 1 \\ 3 & -4 & 5 \\ 5 & -6 & 0 \end{bmatrix}$$

has $\lambda_1 = -1.74$ and $\lambda_{2,3} = -2.63 \pm 3.86\imath$ with eigenvectors

$$v_1 = \begin{bmatrix} .78 \\ .59 \\ -.2 \end{bmatrix}, \qquad v_{2,3} = \begin{bmatrix} -.3 \\ .04 \\ .68 \end{bmatrix} \pm \begin{bmatrix} .23 \\ .63 \\ 0 \end{bmatrix} \imath$$

**Stability:**

- The system is stable because all eigenvalues have negative real part.

**Oscillation:**

- The system will oscillate about the direction

$$\mathrm{Re}(v_{2,3}) = \begin{bmatrix} -.3 \\ .04 \\ .68 \end{bmatrix}$$

For this course, you need to know:

Matrix Analysis:

- eigenvalues and eigenvectors

Dynamical Systems:

- Differential Equations
- Eigenvalues, eigenvectors and the characteristic equation
- State-Space

# You Will Be Responsible For All This Material Throughout the Class!!!

## Any Questions?

# Next Class: Aircraft Dynamics

## The Body-Fixed Frame and Roll-Pitch-Yaw

Next time, we will learn about:
The different frames of reference used for aircraft. This will:

- Define the variables of interest. (e.g. Yaw-Pitch-Roll)
- Determine how we construct our equations of motion.
- Allow us to convert from one frame to another.

Lift and Pitching Moment. This will:

- Develop a framework for writing equations of motion.