Spacecraft and Aircraft Dynamics

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Lecture 2: Coordinate and Variables for Defining the Equations of Motion

This textbook is outlined as follows:

- Chapter 1: Background
- Chapter 2: Force Contributions
  - Wings, Tail, engine, etc.
  - Static Stability (in the body-fixed frame)
- Chapter 3: Non-Equilibrium Equations of Motion
  - Accounts for rotation of the body-fixed frame.
- Chapter 4: Longitudinal Modes
  - Motion in x-z plane.
- Chapter 5: Lateral Modes
  - Motion in y-z plane.

We won't cover chapters 6+

Also included in the text are

- Appendix A: A table of atmospheric properties vs. altitude which has
  - Pressure
  - Temperature
  - density
  - etc.
- Appendix B: Properties of Certain aircraft to be used in the homework problems. Some aircraft have more data listed than others. The aircraft are:
  - NAVION General Aviation Aircraft
  - F104-A Fighter Aircraft
  - A-4D Fighter Aircraft
  - Jetstar Executive Jet
  - Convair 880 Transport
  - Boeing 747 Passenger Aircraft
  - STOL Transport
- Homework problems. Many have errors. If one of the assignments has an undetected error, please alert me.
- Appendix C: Mathematical Review

Aircraft Dynamics

In this section, we will discuss

Reference Frames:

- Body-Fixed Frame
  - A convenient frame for defining forces
- Roll, Pitch and Yaw
  - Relates moments on the aircraft to motion of the aircraft

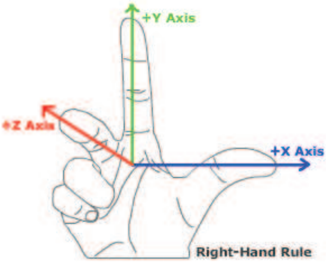
Some Important Angles:

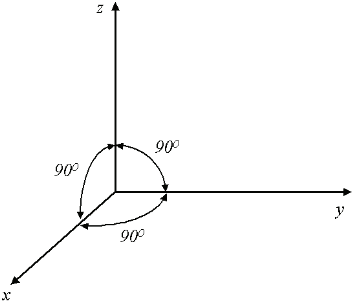
- Angle of attack
- Sideslip

Coordinate Systems

A coordinate system
- defines position variables
- defines positivity

A coordinate system may be
- inertial
  - $F = ma$
- translating
- rotating

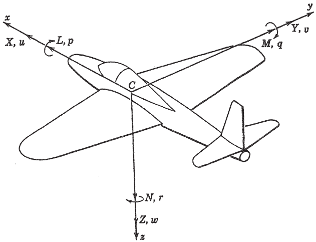


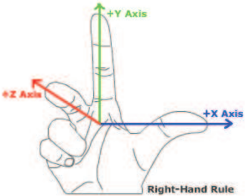


A cartesian coordinate system has right angles and is right-handed.

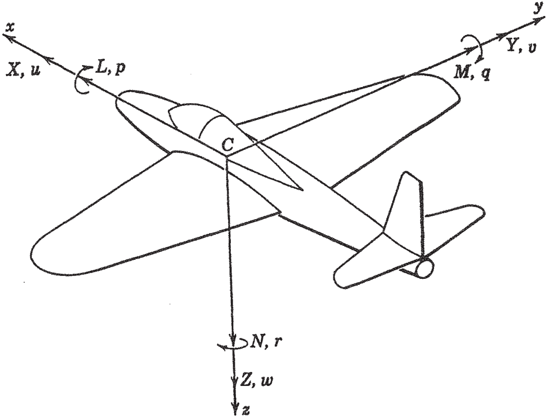
The Body-Fixed Frame

- The origin is the center of mass.
- The x-axis points toward the front of the aircraft.
- The z-axis points down.
- The y-axis is perpendicular to the $x - z$ plane.
- Use the "right-hand rule" to define y





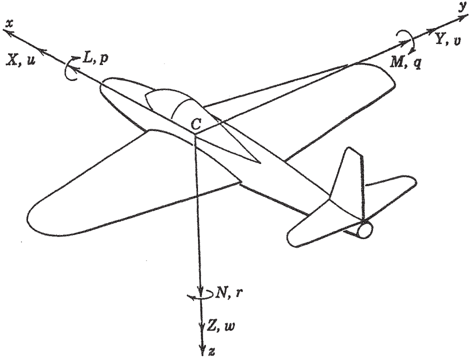
For an idealized aircraft, we might have



Tip of vertical stabilizer	$x = -5$, $z = -1$, $y = 0$.
Tip of the left wing	$x = -1$, $y = -3$, $z = 0$.
Tip of nose	$x = 4$, $y = 0$, $z = 0$

Coordinate Rotations

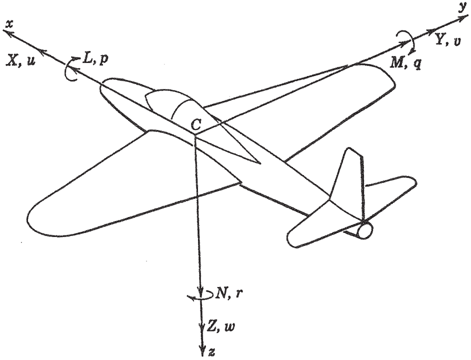
Roll-Pitch-Yaw



There are 3 basic rotations an aircraft can make:

- Roll = Rotation about x-axis
- Pitch = Rotation about y-axis
- Yaw = Rotation about z-axis
- Each rotation is a one-dimensional transformation.

Any two coordinate systems can be related by a sequence of 3 rotations of the



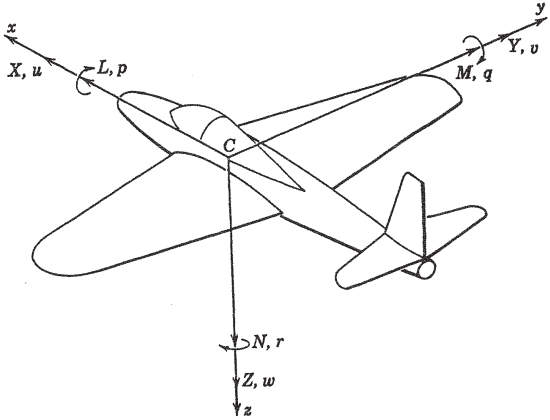
In this diagram, the rate of rotations are labeled.

- $p =$ Rotation rate about x-axis (rad/s) - roll rate
- $q =$ Rotation rate about y-axis (rad/s) - pitch rate
- $r =$ Rotation rate about z-axis (rad/s) - yaw rate

Note bene: Pay careful attention to which direction is positive!!!

Coordinate Rotations

Roll-Pitch-Yaw rates

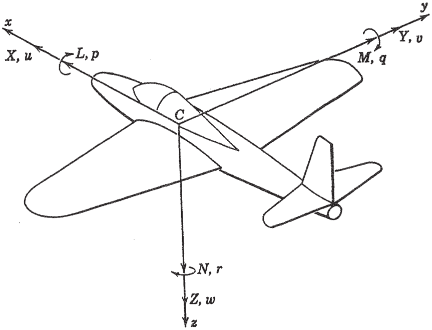


In this diagram,

- A Roll to the *right* is a positive rotation.
- An *upward* pitch is positive.
- A yaw to the *right* is positive.

Forces and Moments

Forces



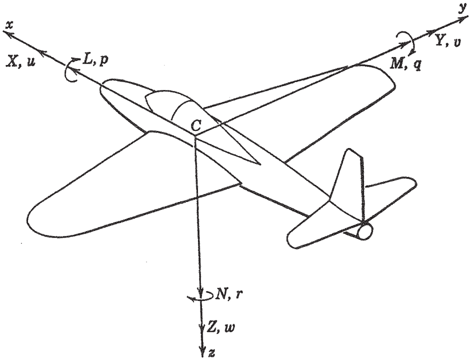
In chapter 1 and 2, we will be concerned with identifying all the forces which act in the body-fixed frame.

These forces and moments have standard labels. The Forces are:

X	Axial Force	Net Force in the positive x-direction
Y	Side Force	Net Force in the positive y-direction
Z	Normal Force	Net Force in the positive z-direction

Forces and Moments

Moments



The Moments are called, intuitively:

L	Rolling Moment	Net Moment in the positive p-direction
M	Pitching Moment	Net Moment in the positive q-direction
N	Yawing Moment	Net Moment in the positive r-direction

Non-dimensional Forces and Moments

Non-dimensional Forces

In the next section, we will see that most of these forces scale in a linear way with something called *Dynamic Pressure*.

Dynamic Pressure:
Dynamic Pressure, Q, refers the pressure of the air moving over the aircraft and is given by

$$Q = \frac{1}{2}\rho v^2$$

where

- ρ is the density of the air (available from Appendix A)
  - kg/m^3 or $slug/ft^3$
  - be careful about slugs!!!
- v is the magnitude of the velocity of the aircraft with respect to the air.
  - m/s or ft/s

Non-dimensional Forces and Moments

Non-dimensional Forces

Among other things, Lift is usually proportional to dynamic pressure. Something like

$$Lift = C_L Q S$$

where

- C_L is a non-dimensional lift coefficient which depends primarily on the airplane configuration and angle-of-attack
- S is surface area of the plane (or another reference area).

Actually, all the forces and moments seem to be roughly proportional to static pressure

- Why????

In any case, this provides a convenient way to quantify the forces and moments without having to account for the effect of altitude and airspeed.

$X = C_x Q S$	Axial Force	Net Force in the positive x-direction
$Y = C_y Q S$	Side Force	Net Force in the positive y-direction
$Z = C_z Q S$	Normal Force	Net Force in the positive z-direction

Thus the forces on the aircraft are defined by the quantities C_x, C_y, and C_z.

Moments are similarly defined

$L = C_l Q S l_w$	Rolling Moment	Net Moment in the positive p-direction
$M = C_m Q S l_w$	Pitching Moment	Net Moment in the positive q-direction
$N = C_n Q S l_c$	Yawing Moment	Net Moment in the positive r-direction

where

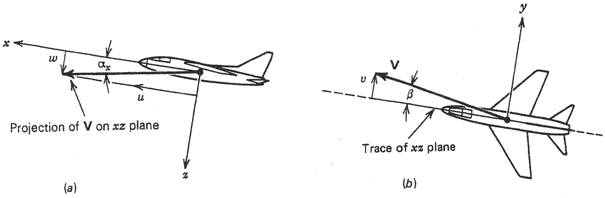
- S again is surface area of the plane (or another reference area).
- l_w is the wingspan
- l_c is the mean chord (to be defined later)

The next chapter will show how to find the coefficients C_x, C_y, C_z, C_l, C_m, and C_n.

Note: Once we have C_x, C_y, C_z, C_l, C_m, and C_n, we still need to account for rotation of the body-fixed frame before finding the equations of motion.

Sideslip and Angle of Attack

Two quantities which heavily influence C_x, C_y, C_z, C_l, C_m, and C_n are angle of attack, α, and sideslip angle, β.

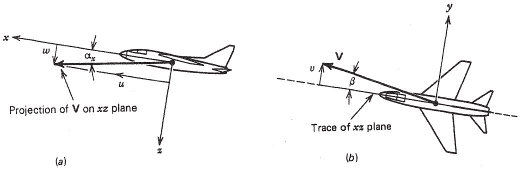


These two angles account for the fact that the nose is not always pointed into the wind.

Thus

- Let $\vec{\mathbf{v}}$ be the velocity vector of the aircraft with respect to the free-stream and expressed in the body-fixed frame.
- If $\vec{\mathbf{v}}$ is projected onto the x-z plane, then α is the angle between the x-axis and this projection.
- If $\vec{\mathbf{v}}$ is projected onto the x-y plane, then β is approximately the angle between the x-axis and this projection.

Sideslip and Angle of Attack



If we introduce the notation

$u = \vec{\mathbf{v}} \cdot \vec{x}$	velocity in the $\vec{x}$ direction
$v = \vec{\mathbf{v}} \cdot \vec{y}$	velocity in the $\vec{y}$ direction
$w = \vec{\mathbf{v}} \cdot \vec{z}$	velocity in the $\vec{z}$ direction

Then α and β can be quantified as

$$\alpha = \tan^{-1} \frac{w}{u} \qquad \text{and} \qquad \beta = \sin^{-1} \frac{v}{V}$$

Where V is the magnitude of the velocity: $V = \sqrt{u^2 + v^2 + w^2}$.

In radians, this approximates as:

$$\alpha \cong \frac{w}{u} \qquad \text{and} \qquad \beta \cong \frac{v}{u}$$

Sideslip and Angle of Attack
Example

Suppose an airplane is flying at $20\,km$ at a speed of $200\,m/s$. The surface area is $30\,m^2$. The aircraft has a wingspan of $10\,m$. Suppose we have the following data

$$C_x = 1.1 \qquad \text{and} \qquad C_y = 0.1 \qquad \text{and} \qquad C_z = 2.3$$

From the atmospheric data in Appendix A, the density of air is $.08891\,kg/m^3$. Then the dynamic pressure is

$$Q = \frac{1}{2}\rho V^2 = \frac{1}{2}.00891 * 200^2 = 178\frac{kg}{ms^2} = 178\frac{N}{m^2}$$

The Lift force is about

$$Z = C_z * Q * S = 2.3 * 178 * 30 = 12,282N$$

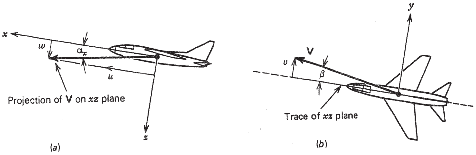
Likewise, the drag is about

$$X = C_x * Q * S = 1.1 * 178 * 30 = 5,874N$$

This gives a Lift-to-Drag ration of about

$$L/D = C_z/C_x = 2.1$$

The pitching moments can be determined in the same way.



Now suppose that the wind hits the aircraft from the direction

$$\vec{v} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 180 \\ 10 \\ 86.6 \end{bmatrix} m/s$$

Then we can find angle of attack and sideslip as approximately

$$\alpha \cong \frac{w}{u} = .48rad = 27.5\deg \qquad \text{and} \qquad \beta \cong \frac{v}{u} = .055rad = 3.18\deg$$

Which contrast with the exact values of

$$\alpha = \tan^{-1}\frac{w}{u} = 25.7\deg \qquad \text{and} \qquad \beta = \sin^{-1}\frac{v}{V} = 2.866\deg$$

Summary

In this lecture, we have covered:

- The body-fixed reference frame.
- The forces and moments acting in the body-fixed frame
- Roll - Pitch - Yaw angles and rates
- Angle of attack
- Sideslip

Next Lecture: Stability

In the next lecture, we will cover;

- Nomenclature
  - Airfoil sections, chord, camber, etc.
- Static Stability
  - Definition of static stability
  - Types of static stability
- Static Longitudinal Stability
  - Conditions for static longitudinal stability