Spacecraft and Aircraft Dynamics

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Lecture 3: Airfoils, and Static Stability

In this lecture, we will discuss

Airfoils:

- Nomenclature
  - Chords, Camber, Aerodynamic Center, etc.
- Forces and Moments produced by lifting surfaces
  - Lift and Moment coefficients and how to interpret them.

Static Stability:

- Definition
- How to determine static longitudinal stability

Next Subject: Lifting Surfaces

How do they work???

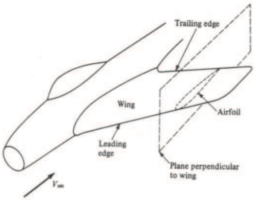


Figure: Airfoil as Cross-Section

Many of the Forces on an aircraft are produced by *Lifting Surfaces*.

- Major exception is propulsion.

Lifting surfaces are characterized by

Planform Shape: The shape of the wing when viewed from above.

- Surface area, tapering, etc.
- determines magnitude of forces.

Airfoil: The cross-section of the wing.

- Determines type of forces and moment.
  - Positivity, location, etc.

Planform Shape: The planform shape of the wing will affect

- Lift
- Drag
- Moment

Surface Area: As mentioned earlier, forces and moments are proportional to.

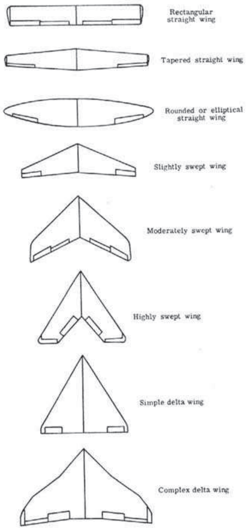
$$F \cong C_L Q S$$

and

$$M \cong C_M Q S l$$

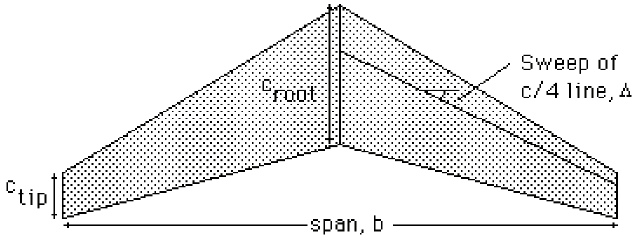
Thus

- Larger wings produce more lift and drag
- Larger wings produce more Moment
- More on chord length, l, shortly



In this course, we will assume a rectangular wing planform.

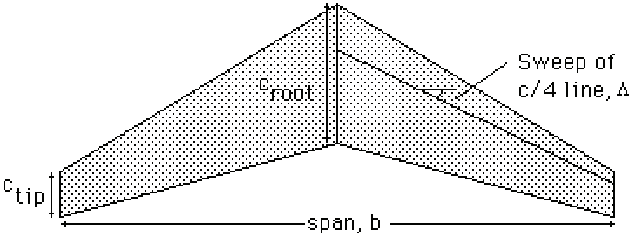
Correction factors can be used for rounded or swept-wing configurations.



Chord, c**:** The width of the surface at some point. This determines the size of the airfoil.

Root Chord, c_r**:** The width of the surface where joined to the airplane.

Tip Chord, c_t**:** The width of the surface at free-stream.



Span, b**:** The total length of the surface.

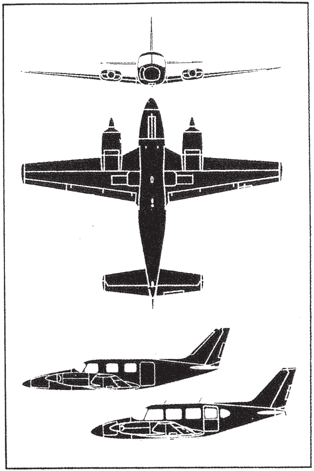
Quarter-Chord Line: The line connecting the points of $1/4$ chord along the span of the surface. The $1/4$ chord point is approximately the aerodynamic center of an airfoil - to be discussed

Sweep: The angle the $1/4$-chord line makes with the horizontal.

Piper PA-31 Navajo USA

Type: light sports aircraft

Accommodation: two pilots, four passengers



Dimensions:
Length: 32 ft 7 in (9.9 m)
Wingspan: 40 ft 8 in (12.4 m)
Height: 13 ft (3.9 m)

Weights:
Empty: 3991 lb (1810 kg)
Max T/O: 6500 lb (2948 kg)

Payload: 350 lb (159 kg)

Performance:
Max speed: 261 mph (420 kmh)
Range: 1065 nm (1973 km)
Power plant: two Lycoming TIO-540-A2C piston engines

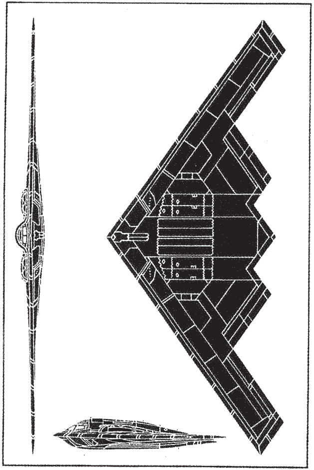
Thrust: 620 hp (231 kW)

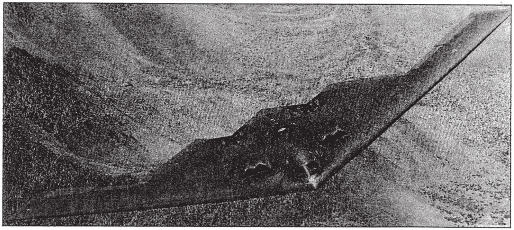
Variants:
Navajo C/R; Pressurized Navajo; Navajo Chieftain stretched version

Notes: First flown in 1964, the Navajo has gone through a number of modifications, the Navajo Chieftain being the most recognizable.

Northrop B-2 Spirit USA

Type: stealth bomber **Accommodation:** one pilot one mission commander side-by-side





Dimensions:
Length: 69ft (21.03m)
Wingspan: 172ft (52.43m)
Height: 17ft (5.18m)

Weights:
Empty: 110 000lb (49 900kg)
Max T/O: 376 000lb (170 550kg)

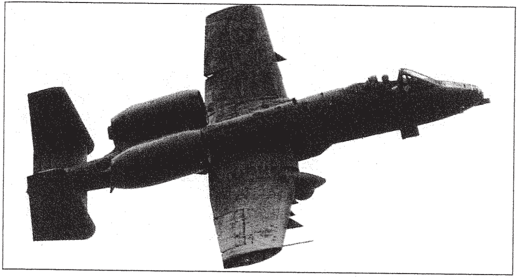
Performance:
Max Speed: n\a
Range: 6600nm (12 223km), 4500nm (8 334km) low level
Powerplant: four General Electric F118-GE-110 turbofans
Thrust: 76 000lb (169kN)

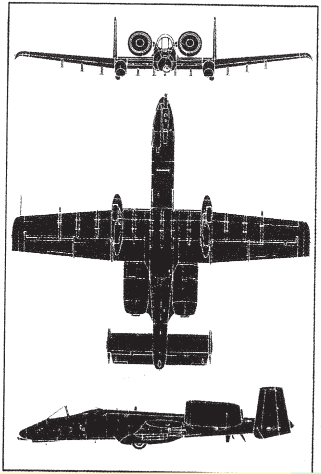
Armament:
internal bomb bay with rotary launchers carrying 16 nuclear weapons or bomb racks carrying conventional weapons; 50 000lb (22 680kg) warload; AGM-131 SRAM II stand-off nuclear weapon, AGM-129 nuclear cruise missile; B61, B83 free fall nuclear weapons; conventional bombs and mines

Notes: The most expensive aircraft ever running at $865 million per aircraft. The USAF originally wanted 133 B-2s, but only 20 had been funded by 1995. There is provision for a third crew member behind the commander.

Fairchild A-10A Thunderbolt USA

Type: close support aircraft **Accommodation:** one pilot





Dimensions:
Length: 53 ft 4 in (16.26 m)
Wingspan: 57 ft 6 in (17.53 m)
Height: 14 ft 8 in (4.47 m)

Weights:
Empty: 23 370 lb (10 710 kg)
Max T/O: 47 400 lb (21 500 kg)

Performance:
Max Speed: 449 mph (722 kmh)
Range: 1080 nm (2000 km)
Powerplant: two General Electric TF34-GE-100 high bypass ratio turbofans
Thrust: 18 130 lb (80.6 kN)

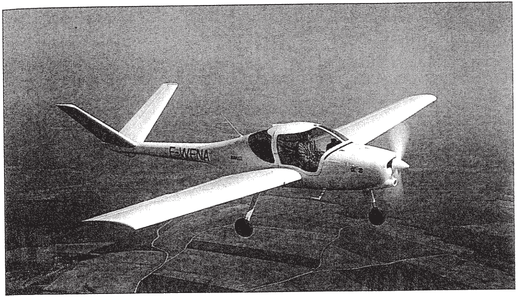
Armament:
one 30 mm GAU-8/A seven-barrelled cannon; 11 hardpoints; 16 000 lb (7257 kg) warload; AGM-65A Maverick; wide range of bombs

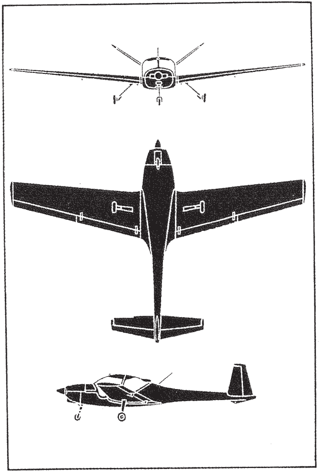
Variants:
OA-10A Fast FAC aircraft

Notes: The pilot is protected by a titanium `bathtub' capable of withstanding 23 mm gun fire.

Robin ATL Club France

Type: light sports aircraft
Accommodation: two pilots





Dimensions:
Length: 22 ft (6.7 m)
Wingspan: 33 ft 7 in (10.2 m)
Height: 6 ft-6 in (2 m)

Weights:
Empty: 794 lb (360 kg)
Max T/O: 1278 lb (580 kg)
Payload: n\a

Performance:
Max speed: 124 mph (200 kmh)
Range: 539 nm (1000 km)

Power plant: one JPX 4T 60A piston engine
Thrust: 65 hp (48 kW)

Variants:
Model 88 improved version; re-engined Model 89

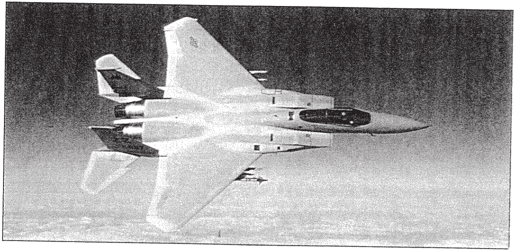
Notes: First flown in 1983, the ATL Club is powered by a converted Volkswagen car engine.

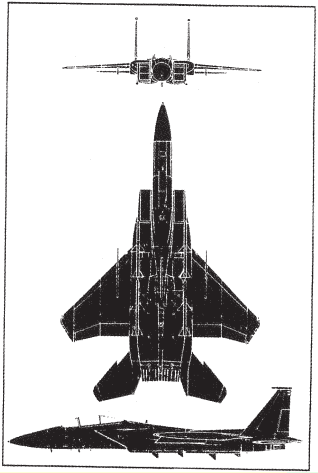
Lifting Surfaces

Examples

McDonnell Douglas F-15C Eagle USA

Type: air superiority fighter **Accommodation:** one pilot





Dimensions:
Length: 63 ft 9 in (19.43 m)
Wingspan: 42 ft 9 in (13.05 m)
Height: 18 ft 5 in (5.63 m)

Weights:
Empty: 28 600 lb (12 973 kg)
Max T/O: 68 000 lb (30 845 kg)

Performance:
Max Speed: Mach 2.5+
Range: 2500 nm (4631 km)
Powerplant: two Pratt & Whitney F100-PW-220 turbofans
Thrust: 47 540 lb (211.4 kN) with afterburner

Armament:
one 20 mm M61A1 Vulcan cannon; 11 hardpoints; four AIM-7 Sparrow or AIM-120 AMRAAM; four AIM-9 Sidewinder

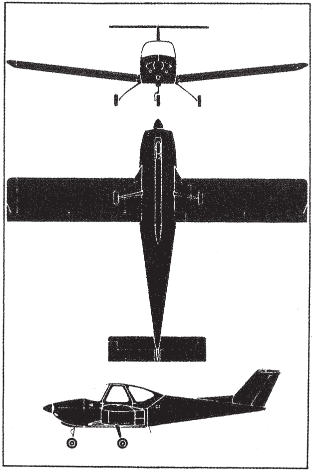
Variants:
F-15D twin-seat operational trainer
F-15J version for Japan
F-15DJ two-seater for Japan

Notes: Can be configured to carry conformal fuel tanks and extra ECM kit

Beechcraft Skipper USA

Type: light training aircraft

Accommodation: two pilots, one passenger



Dimensions:
Length: 23 ft 10 in (7.3 m)
Wingspan: 30 ft (9.1 m)
Height: 7 ft 6 in (2.3 m)

Weights:
Empty: 1103 lb (500 kg)
Max T/O: 1650 lb (748 kg)
Payload: n\a

Performance:
Max speed: 120 mph (196 kmh)
Range: 413 nm (764 km)
Power plant: one O-235 piston engine
Thrust: 115 hp (85 kw)

Variants:
none

Notes: Built in small numbers, the Skipper is mainly used for training purposes.

Lifting Surfaces
Airfoils

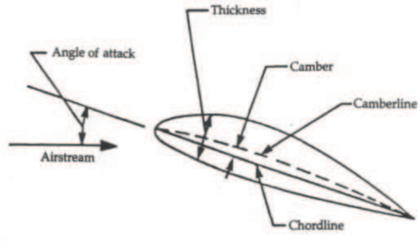


Figure: Airfoil with positive camber

Chord Line: A line connecting the leading edge to the trailing edge.

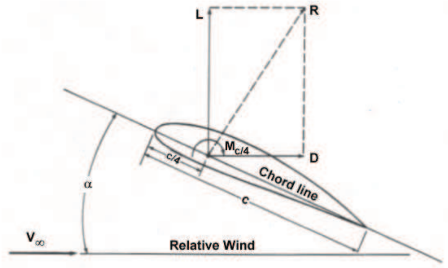
Camber Line: A line connecting the points halfway between the top and bottom surfaces.

Camber: Camber refers to the difference between the chord line and the camber line. Camber determined the moment produced by a wing. Most wings have positive camber.

Airfoils: Aerodynamic Center

Aerodynamic Center: The point at which the pitching moment does not vary with angle of attack.

- Convenient since C_M is now static.
- Typically located at the $1/4$-chord line.



Forces and Moments: The motion of air creates forces and moments.

- Lift and Drag are measured at the *aerodynamic center*.
- Moment is measured as the moment about the *aerodynamic center*.
- Usually take standard form

$$L = C_L Q S, \qquad D = C_D Q S, \qquad \text{and} \qquad M = C_M Q S l$$

- C_L and C_D will depend on angle of attack and airfoil geometry.
- C_M will (hopefully) depend only on airfoil geometry, especially camber.

Airfoils: Lift Coefficient

Lift is given by

$$L = C_L Q S$$

General Form:

$$C_L = C_{L0} + C_{L\alpha}\alpha$$

where

- C_{L0} is the lift produced at steady-level flight. We define $C_{L0} = 0$ for an airfoil. However, for the aircraft overall, we want $C_{L0} > 0$.
  - Don't want to fly nose-up all the time.
- $C_{L\alpha} > 0$ is determined by the airfoil type and other factors.
  - Sweep, planform shape, winglets, Mach number, etc.

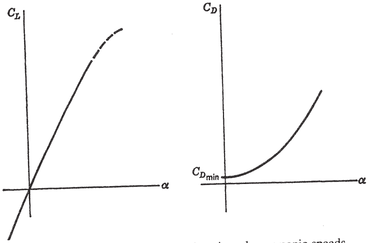


Figure 2.2 Lift and drag for subsonic and supersonic speeds.

Airfoils: Drag Coefficient

$$D = C_D Q S$$

The drag coefficient, C_D, of an airfoil is related to the lift coefficient, C_L. It can be approximated as

$$C_D = C_{D0} + K C_L^2$$

where

- C_{D0} and K are determined by airfoil type and other factors
  - Mach number, thrust coefficient, etc.

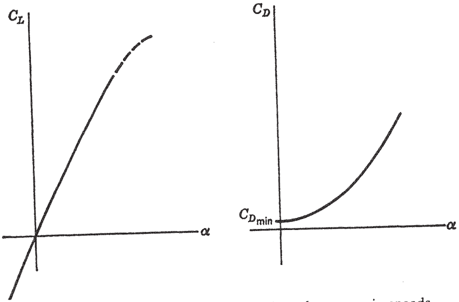


Figure 2.2 Lift and drag for subsonic and supersonic speeds.

Airfoils: Moment Coefficient

Positive pitching moment is given by

$$M = C_M Q S l$$

General Form:

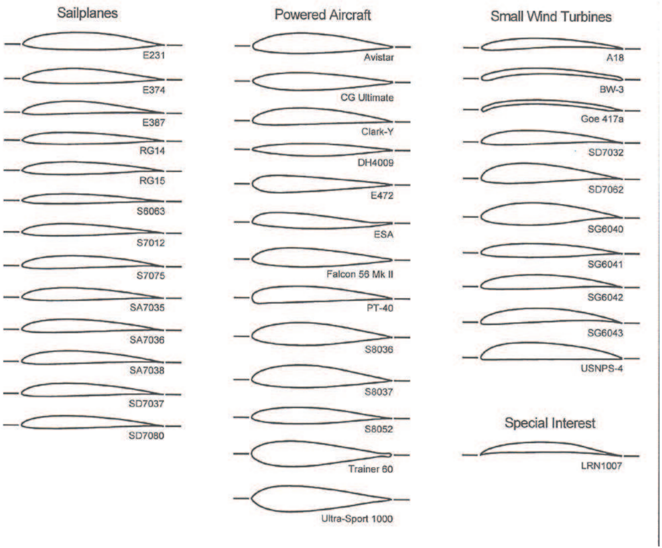
$$C_M = C_{M0} + C_{M\alpha}\alpha$$

where

- C_{M0} is the moment produced at steady-level flight. For an airfoil we have the confusing terminology:
  - $C_{M0} < 0$ for if the airfoil has **positive** camber.
  - $C_{M0} > 0$ for if the airfoil has **negative** camber.
- For the aircraft overall, we typically want $C_{M0} > 0$ (negative camber), but most airfoils have positive camber.
- By definition $C_{M\alpha} = 0$ for an airfoil if we are considering moment about the aerodynamic center. We will next discuss the effect of $C_{M\alpha}$ on the overall airplane.

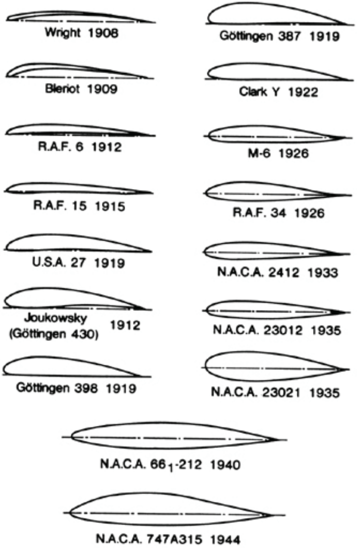
Airfoils: Examples

Low-Speed Airfoils



Airfoils: Examples

Earl Airfoil Evolution



Airfoils: Examples

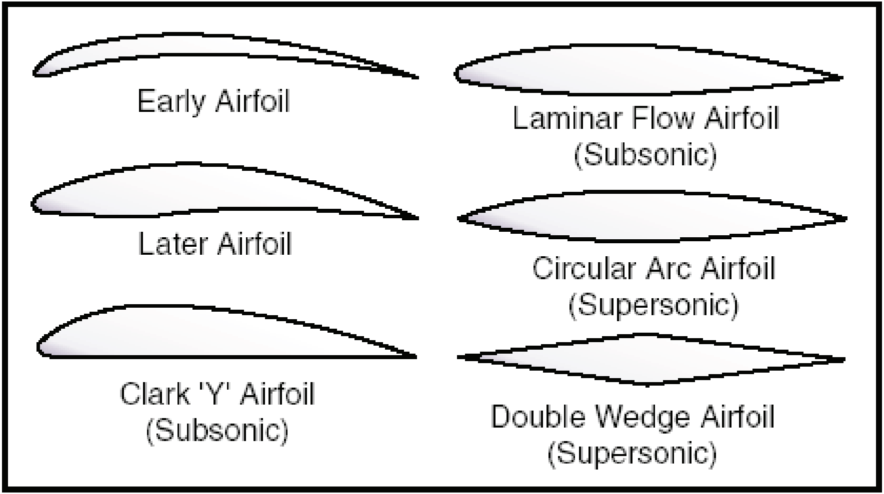


Figure: Later Airfoils

Big Idea: Static Stability

We now introduce the poorly-defined notion of static stability.

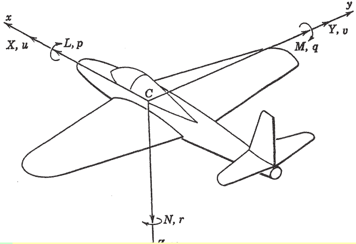
Static Stability is stability of the equations of motion for if we only consider 2 dimensions and consider the body-fixed frame to be inertial.

We have the following three varieties, in order of interest:

Static Longitudinal Stability: “Pseudo-stability” of the pitching dynamics. Only motion in the q-direction.

Static Directional Stability: “Pseudo-stability” of the yawing dynamics. Only motion in the r-direction.

Static Roll Stability: “Pseudo-stability” of the roll dynamics. Only motion in the p-direction.



Longitudinal or pitching stability is the most common question we consider. The dynamics are very simple

$$M = I\dot{q} = I\ddot{\alpha} = (C_{M0} + C_{M\alpha}\alpha)QSl$$

where

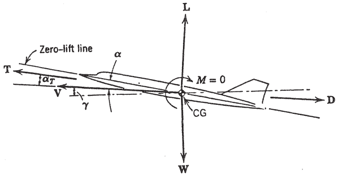
- I is a moment of inertia term.
- Recall $q = \dot{\alpha}$

Equilibrium occurs when

$$\dot{q} = \ddot{\alpha} = (C_{M0} + C_{M\alpha}\alpha)\frac{QSl}{I} = 0.$$

Thus the equilibrium point is

$$\alpha = -\frac{C_{M0}}{C_{M\alpha}}.$$



Static Longitudinal Stability: Conceptual Description

Nose-Up Configuration

Regarding the equilibrium point

$$\alpha_E = -\frac{C_{M0}}{C_{M\alpha}}.$$

- Typically, we prefer an aircraft with *nose-up* in steady-level flight.
  - An aircraft is "nose-up" when $\alpha > 0$.
- Steady-level flight means an aircraft in equilibrium.

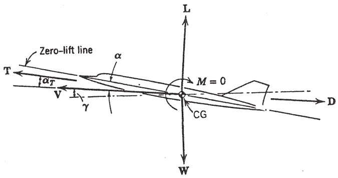
Thus, when designing an aircraft, we want $\alpha_E > 0$. This is achieved when

Case 1:

- $C_{M0} \geq 0$
- $C_{M\alpha} \leq 0$

Case 2:

- $C_{M0} \leq 0$
- $C_{M\alpha} \geq 0$



In the next slide we will show that for stability, only Case 1 is possible.

For a given equilibrium, the aircraft is

- Stable if a positive displacement results in negative restoring force.
- Unstable if a positive displacement results in a positive force.

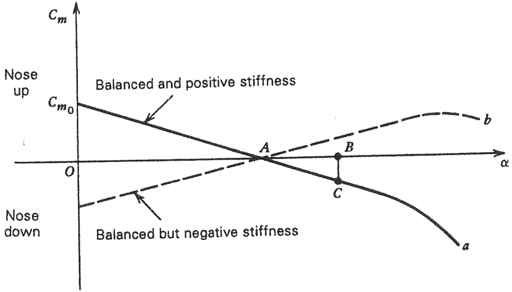


Figure 2.3 Pitching moment of an airplane about the CG.

Static Longitudinal Stability

Dynamics about the Equilibrium

Stability is about displacement from equilibrium:

$$\Delta\alpha(t) = \alpha(t) - \alpha_E$$

Stability Question: Does $\lim_{t\to\infty} \Delta\alpha(t) \to 0$???
The dynamics of the displacement, $\Delta\alpha$ are

$$\begin{aligned}
\frac{d^2}{dt^2}\Delta\alpha(t) &= \frac{d^2}{dt^2}\alpha(t) - \frac{d^2}{dt^2}\alpha_E = \ddot{\alpha}(t) \\
&= (C_{M0} + C_{M\alpha}\alpha(t))\frac{QSl}{I} \\
&= (C_{M0} + C_{M\alpha}\left(\Delta\alpha(t) + \alpha_E\right))\frac{QSl}{I} \\
&= (C_{M0} - C_{M\alpha}\frac{C_{M0}}{C_{M\alpha}} + C_{M\alpha}\Delta\alpha(t))\frac{QSl}{I} \\
&= C_{M\alpha}\frac{QSl}{I}\Delta\alpha(t).
\end{aligned}$$

Stability *always* considers displacement from equilibrium!!!

Static Longitudinal Stability

Characteristic Equation

The displacement dynamics are

$$\frac{d^2}{dt^2}\Delta\alpha(t) = C_{M\alpha}\frac{QSl}{I}\Delta\alpha(t)$$

Thus the characteristic equation is $s^2 - \frac{C_{M\alpha}QSl}{I}$, which has roots at

$$s_{1,2} = \pm\frac{1}{2}\sqrt{\frac{C_{M\alpha}QSl}{I}}$$

- We want to know if any roots have positive real part.

Since Q, S, l and I are all positive, there are two cases.

Case 1:

- $C_{M\alpha} \geq 0$

Aircraft is Unstable!

Case 2:

- $C_{M\alpha} < 0$

Aircraft is Stable, but oscillates.

Sometimes, dC_M/dC_L data is used instead of $C_{M\alpha} = dC_M/d\alpha$. This can be done because

$$C_L = C_{L0} + C_{L\alpha}\alpha, \qquad \text{so} \qquad \frac{dC_L}{d\alpha} = C_{L\alpha}.$$

Therefore,

$$C_{M\alpha} = \frac{dC_M}{d\alpha} = \frac{dC_M}{dC_L} * \left(\frac{dC_L}{d\alpha}\right)^{-1} = \frac{dC_M}{dC_L} C_{L\alpha}$$

Since $C_{L\alpha}$ is constant and positive, we have

- **Stability** if $\frac{dC_M}{dC_L} < 0$
- **Instability** if $\frac{dC_M}{dC_L} \geq 0$

This is useful if we also want to balance Lift and weight.

An aircraft without a tail has the following moment characteristics:

$$C_{M0} = -.4 \qquad \text{and} \qquad C_{M\alpha} = -.2\,\mathrm{deg}^{-1}.$$

Describe the steady-state motion.

- $C_{M\alpha} = -.2\,\mathrm{deg}^{-1}$, so aircraft is stable.
- $C_{M0} = -.4$, so equilibrium is at

$$\alpha_E = -\frac{C_{M0}}{C_{M\alpha}} = -\frac{-.4}{-.2}\,\mathrm{deg} = -2\,\mathrm{deg}$$

So Equilibrium is Nose Down! The plane will not produce enough lift.

Question: How do we fix the airplane???
Answer: Add a tail.
Question: Alternatives??? Flaps, inclined wings, etc.

Aircraft Incidents: Failure Modes

Short-Period Failure Mode in SAAB Grippen

Aircraft Incidents: Failure Modes

Long-Period? Failure Mode in SAAB Grippen

Aircraft Incidents: Failure Modes

F-22 Longitudinal Mode

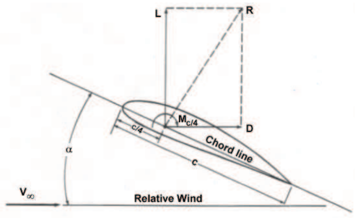
Aircraft Incidents: Failure Modes

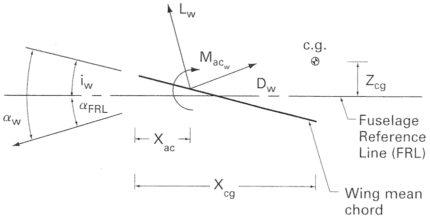
F-8 Pilot-Induced Oscillation

Aircraft Incidents: Failure Modes

Space Shuttle Pilot-Induced Oscillation

Two Confusing Figures





Confusion: For an airfoil, angle of attack is measured to the zero-lift-line.

- Thus $C_{M0} = 0$ for an un-inclined *airfoil*.

Confusion: We assume that the aerodynamic center is on the FRL.

- Thus as measure from the CG,

$$\vec{r}_{ac} = \begin{bmatrix} X_{cg} - X_{ac} \\ 0 \\ Z_c g \end{bmatrix}.$$

- If there is any confusion on a problem, ask me to clarify.

To summarize these two results:

To have static longitudinal stability, we need

- $C_{M\alpha} \leq 0$

To have longitudinal stability **AND** nose-up in steady state, we need

- $C_{M0} \geq 0$

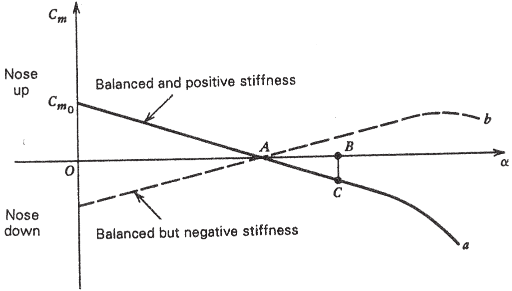


Figure 2.3 Pitching moment of an airplane about the CG.

Next Lecture: Finding C_{M0} and $C_{M\alpha}$

The C_{M0} and $C_{M\alpha}$ of an airplane are determined by adding up the contributions of all factors.

In the next lecture, we will discuss the contributions of

- Rectangular Wing
- Horizontal Stabilizer
- Canards