Spacecraft and Aircraft Dynamics

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Lecture 6: Neutral Point and Elevator Control

Aircraft Dynamics

Lecture 6

In this lecture, we will discuss

Neutral Point:

- The location of the CG for which $C_{M\alpha,total} = 0$
- Static Margin

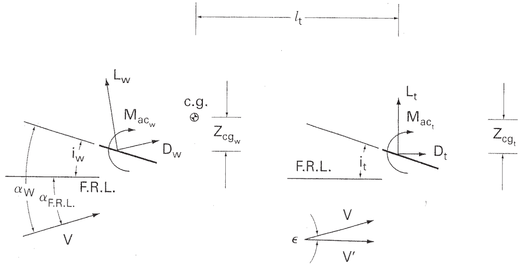
Control Surfaces:

- Elevator deflection
- Trim

Examples:

- Calculating X_{NP}, etc.

Review

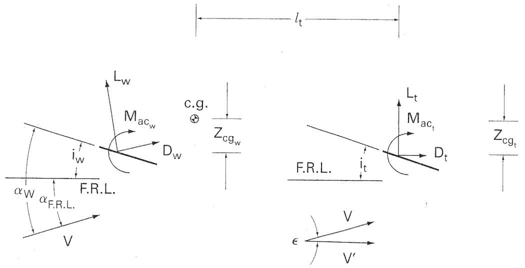


Wing Contribution:

$$C_{M0,w} = C_{M0,w} + C_{L\alpha,w} i_w \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right)$$

and

$$C_{M\alpha,w} = C_{L\alpha,w} \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right) \leq 0$$



Tail Contribution:

$$C_{M0,t} = \eta V_H C_{L\alpha,t} \left(\varepsilon_0 + i_w - i_t\right)$$

and

$$C_{M\alpha,t} = C_{L\alpha,w} \left(\frac{X_{cg}}{\bar{c}} - \frac{X_{ac}}{\bar{c}} \right) \leq 0$$

$$V_H = \frac{l_t S_t}{S \bar{c}}, \qquad \varepsilon = \frac{2 C_{L0,w}}{\pi A R_w}$$

Now we have a moment equation.

$$C_{M,total} = C_{M0,total} + C_{M\alpha,total}\alpha$$

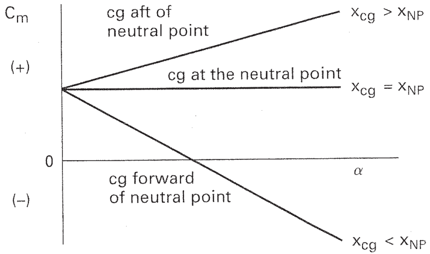
where

$$C_{M0,total} = C_{M0,wf} + \eta V_H C_{L\alpha,t}(\varepsilon_0 + i_{wf} - i_t)$$

$$C_{M\alpha,total} = C_{L\alpha,wf}\left(\frac{X_{CG}}{\bar{c}} - \frac{X_{AC,wf}}{\bar{c}}\right) - \eta V_H C_{L\alpha,t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

Design Aspects:

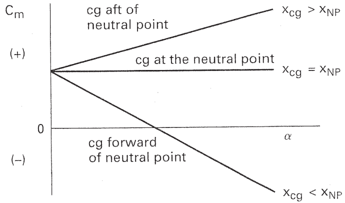
- X_{CG}
- V_H
- i_{wf}, i_t
- ....?



For stability, we need $C_{M\alpha,total} < 0$. How does the location of the CG affect stability?

Question:

- How far aft can we place the CG and retain stability?
  - What is the maximum X_{CG} so that $C_{M\alpha} < 0$.

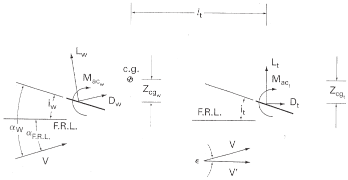


Definition 1.

The **Neutral Point** is the X_{CG} for which $C_{M\alpha,total} = 0$.

For wing+tail, $C_{M\alpha}$ has the form

$$
\begin{aligned}
C_{M\alpha} &= C_{L\alpha}\left(X_{CG} - X_{AC}\right)/\bar{c} - \eta V_H C_{L\alpha,t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \\
&= C_{L\alpha}\frac{X_{CG}}{\bar{c}} - C_{L\alpha}\frac{X_{AC}}{\bar{c}} - \eta V_H C_{L\alpha,t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)
\end{aligned}
$$

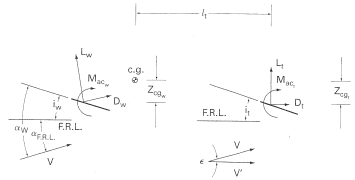


To find the $X_{NP} = \max_{C_{M\alpha>0}} X_{CG}$, we must solve the equation

$$C_{M\alpha} = C_{L\alpha}\frac{X_{NP}}{\bar{c}} - C_{L\alpha}\frac{X_{AC}}{\bar{c}} - \eta V_H C_{L\alpha,t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) = 0$$

for X_{CG}. This solution is given by

$$X_{NP} = X_{AC} + \bar{c}\eta V_H \frac{C_{L\alpha,t}}{C_{L\alpha}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$



Definition 2.

The **Static Margin** is the normalized distance between the Neutral Point and the CG.

$$K_n := \frac{X_{NP}}{\bar{c}} - \frac{X_{CG}}{\bar{c}}$$

By definition, $K_n > 0$ for a stable aircraft.

Recall from the solution of X_{np}

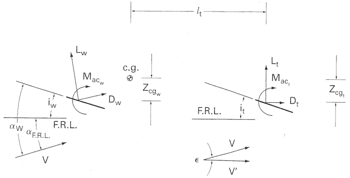
$$\frac{X_{np}}{\bar{c}} = \frac{X_{ac}}{\bar{c}} + \eta V_H \frac{C_{L\alpha,t}}{C_{L\alpha}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

and the moment equation,

$$\frac{X_{cg}}{\bar{c}} = \frac{C_{M\alpha}}{C_{L\alpha}} + \frac{X_{ac}}{\bar{c}} + \eta V_H \frac{C_{L\alpha,t}}{C_{L\alpha}}\left(1 - \frac{d\varepsilon}{d\alpha}\right),$$

we have

$$\begin{aligned} K_n &= \frac{X_{NP}}{\bar{c}} - \frac{X_{CG}}{\bar{c}} \\ &= -\frac{C_{M\alpha}}{C_{L\alpha}} \\ &= -\frac{dC_M}{dC_L} \end{aligned}$$



$$
\begin{aligned}
C_L &= C_{Lwf} + \eta \frac{S_t}{S} C_{Lt} = C_{L\alpha,wf}\alpha_{wf} + \eta \frac{S_t}{S} C_{L\alpha,t}\left(\alpha_{wf} - \varepsilon - i_{wf} + i_t\right) \\
&= \left[C_{L\alpha,wf} + \eta \frac{S_t}{S} C_{L\alpha,t}\right]\alpha_{wf} - \left[\eta \frac{S_t}{S} C_{L\alpha,t}\left(\varepsilon + i_{wf} - i_t\right)\right] \\
&= C_{L0,total} + C_{L\alpha,total}\alpha_{wf}
\end{aligned}
$$

where

$$
C_{L0,total} = -\eta \frac{S_t}{S} C_{L\alpha,t}\left(\varepsilon + i_{wf} - i_t\right) \quad C_{L\alpha,total} = \left[C_{L\alpha,wf} + \eta \frac{S_t}{S} C_{L\alpha,t}\right]
$$

Example: Steady-Level Flight

For a given flight condition, we have Q. To satisfy Lift=Weight, we need

$$W = L = (C_{L\alpha}\alpha + C_{L0})QS$$

Therefore, we can solve for

$$\alpha_{wf,d} = \frac{W - C_{L0}QS}{QSC_{L\alpha}}$$

To make α_d the equilibrium point, we must satisfy

$$\alpha_d = -\frac{C_{M0}}{C_{M\alpha}}$$

Process:

1. Locate X_{CG} so that $C_{M\alpha} < 0$.
2. Find S_t, so that
$$C_{M0} = -C_{M\alpha}\alpha_d$$

Question: What happens when we change *Altitude*?

Example: Steady-Level Flight

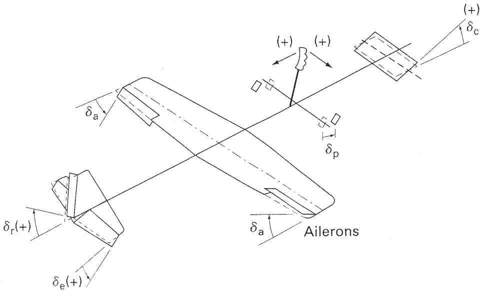
Change in Flight Condition

When we change altitude or velocity, what changes?

- Dynamic pressure changes - Q.
- $C_{M\alpha}$ and C_{M0} are unaffected.
  - Thus, α_{eq} and stability don't change
- $C_L = C_{L\alpha}\alpha$ doesn't change
- Lift
  - *Increases* as we descend.
  - *Decreases* as we ascend.

Conclusion: There is only one stable altitude for steady-level flight?
Question: How can we change altitude?

The lift force on a lifting surface can be modified by means of a *Control Surface*.

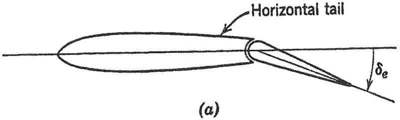


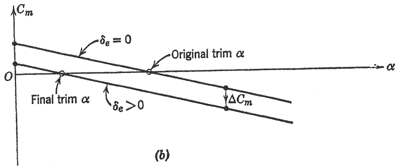
Control Surfaces on the **Wing** are called **Ailerons**

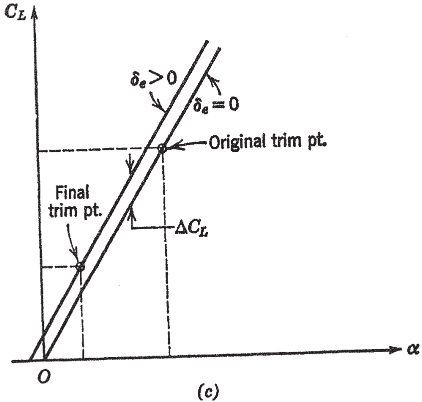
A Control Surface on the **Horizontal Stabilizer** is called an **Elevator**.

A Control Surface on the **Vertical Stabilizer** is called a **Rudder**.

Lets focus on the effect of the **Elevator**.

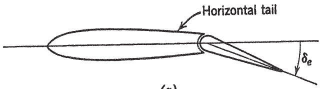






An elevator deflection changes the lift produced by the tail - $C_{L0,t}$

Deflection of the control surface creates an increase or decrease in *Lift*.



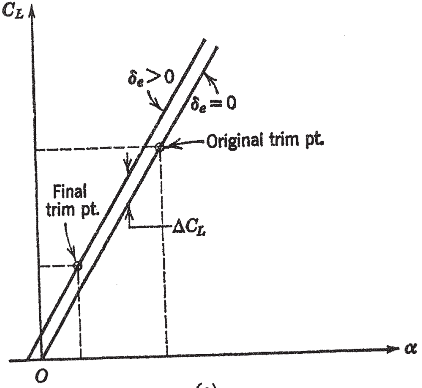
(a)

We quantify the effect by adding a C_{L0}, but leaving $C_{L\alpha}$ unchanged.

$$\Delta C_L = C_{L\delta_e}\delta_e$$

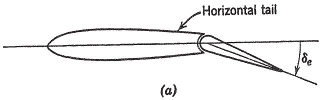
where we define $C_{L\delta_e} := \frac{dC_L}{d\delta_e}$. Then for an isolated airfoil,

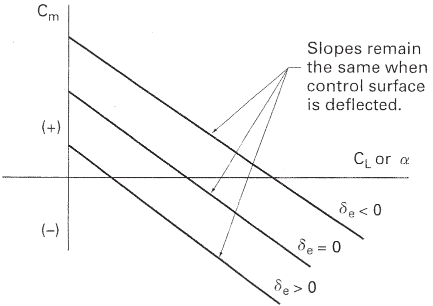
$$C_L = C_{L\delta_e}\delta_e + C_{L\alpha}\alpha$$



(c)

Deflection of the control surface also creates an increase or decrease in *Moment*.



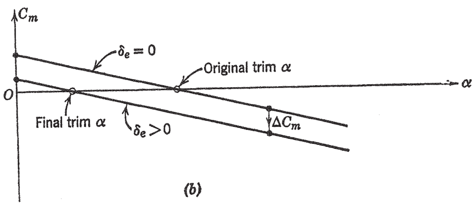


Again, the effect is to add a C_{M0}, but to leave $C_{M\alpha}$ unchanged.

$$\Delta C_M = C_{M\delta_e}\delta_e$$

where $C_{M\delta_e} := \frac{dC_M}{d\delta_e}$. Then for the isolated airfoil,

$$C_M = C_{M\delta_e}\delta_e + C_{M\alpha}\alpha$$



Suppose the line without δ_e corresponds to steady level flight. If we want to increase altitude,

- ρ decreases, so Q decreases.
- We must increase C_L to maintain Lift=Weight.
- We create a negative δ_e on the tail
  - This increases C_{Mt} and decreases C_{Lt}.
- α_{eq} increases, which means $C_{L,total}$ increases.
- Finally, $L = W$

Elevator Deflection

Elevator effectiveness

An elevator effectively acts to change the inclination of the airfoil. Recall

$$\Delta C_L = C_{L\delta_e}\delta_e.$$

We can model $C_{L\delta_e}$ as

$$C_{L\delta_e} = \frac{dC_L}{d\delta_e} = \frac{dC_L}{d\alpha}\frac{d\alpha}{d\delta_e} = C_{L\alpha}\tau$$

Definition 3.

The **Elevator Effectiveness** is defined as

$$\tau = \frac{d\alpha}{d\delta_e}$$

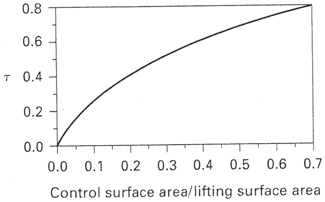
Elevator Effectiveness is primarily determined by

- Surface Area of Elevator/ Surface Area of Tail.

Elevator Deflection

Elevator effectiveness

$$\tau = \frac{d\alpha}{d\delta_e}$$

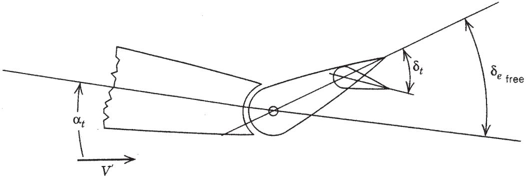


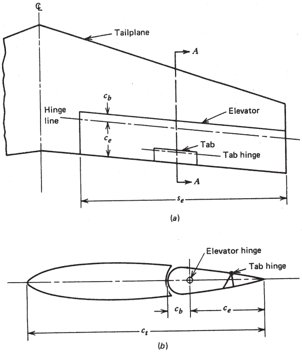
Clearly

- As surface area decreases, $\tau \to 0$.
- As surface area increases, $\tau \to 1$.
- Law of diminishing returns.

The same approach applies to aileron and rudder deflections - To be discussed.

To avoid holding elevators at constant deflection, **Trim Tabs** are often used.





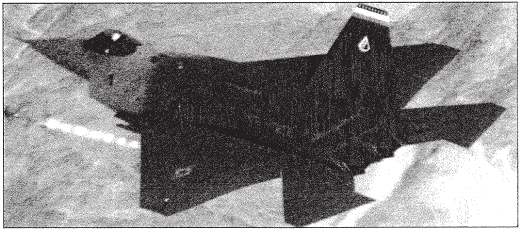
Trim tabs act as elevators for the elevators.
Create a bias proportional to

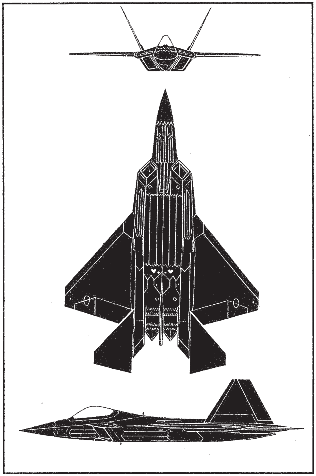
$$\Delta C_L = \tau_t C_{L\alpha,t} \delta_t.$$

Trim tab effectiveness scales in the same manner as elevator effectiveness.

Lockheed/General Dynamics/ Boeing F-22 Lightning II USA

Type: advanced tactical fighter **Accommodation:** one pilot





Dimensions: production model
Length: 62 ft 1 in (18.92 m)
Wingspan: 43 ft 0 in (13.11 m)
Height: 16 ft 5 in (5 m)

Weights:
Empty: YF-22 30 000 lb (13 608 kg)
Max T/O: F-22 60 000 lb (27 216 kg)

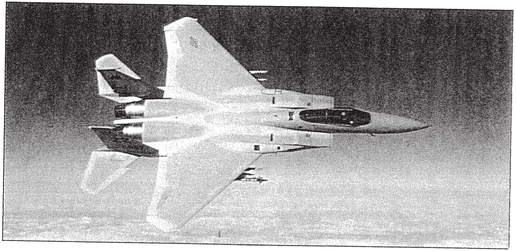
Performance: YF-22
Max Speed: supercruise Mach 1.58 – Mach 1.7 with afterburner
Range: unknown
Powerplant: two Pratt & Whitney F119-PW-100 advanced technology engines with vectoring exhaust nozzles
Thrust: 70 000 lb (310 kN)

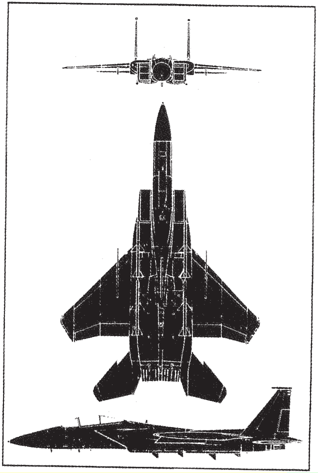
Armament:
one long-barrel 20mm gun; three internal bays, four external hardpoints for ferry tanks only; AIM-120 AMRAAM, AIM-9L Sidewinder AAMs

Variants:
YF-22 prototype and proof of concept aircraft
F-22A enlarged, refined production fighter

McDonnell Douglas F-15C Eagle USA

Type: air superiority fighter **Accommodation:** one pilot





Dimensions:
Length: 63 ft 9 in (19.43 m)
Wingspan: 42 ft 9 in (13.05 m)
Height: 18 ft 5 in (5.63 m)

Weights:
Empty: 28 600 lb (12 973 kg)
Max T/O: 68 000 lb (30 845 kg)

Performance:
Max Speed: Mach 2.5+
Range: 2500 nm (4631 km)
Powerplant: two Pratt & Whitney F100-PW-220 turbofans
Thrust: 47 540 lb (211.4 kN) with afterburner

Armament:
one 20 mm M61A1 Vulcan cannon; 11 hardpoints; four AIM-7 Sparrow or AIM-120 AMRAAM; four AIM-9 Sidewinder

Variants:
F-15D twin-seat operational trainer
F-15J version for Japan
F-15DJ two-seater for Japan

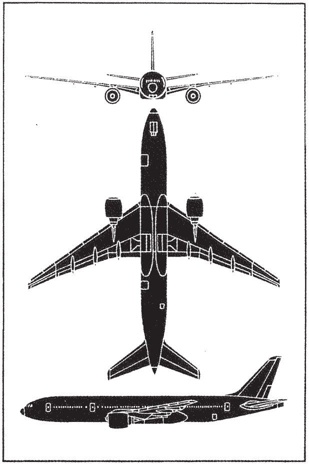
Notes: Can be configured to carry conformal fuel tanks and extra ECM kit

Elevator Effectiveness

Examples

Boeing 777 USA

Type: long-haul widebody airliner **Accommodation:** two pilots; 440 passengers



Dimensions:
Length: 209 ft 1 in (63.7 m)
Wingspan: 199 ft 11 in (60.9 m)
Height: 60 ft 9 in (18.5 m)

Weights:
Empty: 304 500 lb (138 120 kg)
Max T/O: 590 000 lb (267 620 kg)
Payload: 120 500 lb (54 660 kg)

Performance:
Cruising speed: mach 0.87
Range: 7380 nm (13 667 km)
Power plant: two Pratt & Whitney, General Electric or Rolls-Royce turbofans
Thrust: 168 000 lb (747.2 kN)

Variants:
A, market standard version; B, market heavier version

Notes: The latest airliner from Boeing, the 777 is also their first fly-by-wire. The outer 21 ft 3 in (6.5 m) of each wing can be folded upwards to reduce width at airport gates. The launch customer, United Airlines, took delivery of the first 777 in mid-1995.

Elevator Deflection

Total Pitching Moment

The change in total pitching moment is primarily due to changes in Lift. We neglect $\Delta C_{M0,t}$

$$\begin{aligned}\Delta C_{M,total} &= -V_H \eta \Delta C_{Lt} \\ &= -V_H \eta \Delta \tau C_{L\alpha,t} \delta_e\end{aligned}$$

Thus the total pitching moment can be written as

$$C_{M,total} = C_{M0,total} + C_{M\alpha,total}\alpha + C_{M\delta_e,total}\delta_e.$$

Elevator Deflection

Trimming

To alter the flight condition of an aircraft, we

1. Solve $L = W$ to find α_{eq}.

$$\alpha_{eq} = \frac{C_{L,desired} - C_{L,\delta_e}\delta_e}{C_{L,\alpha}}$$

2. At Moment equilibrium, we have

$$C_{M,total} = C_{M0,total} + C_{M\alpha,total}\alpha_{eq} + C_{M\delta_e,total}\delta_e = 0$$

So we can solve for

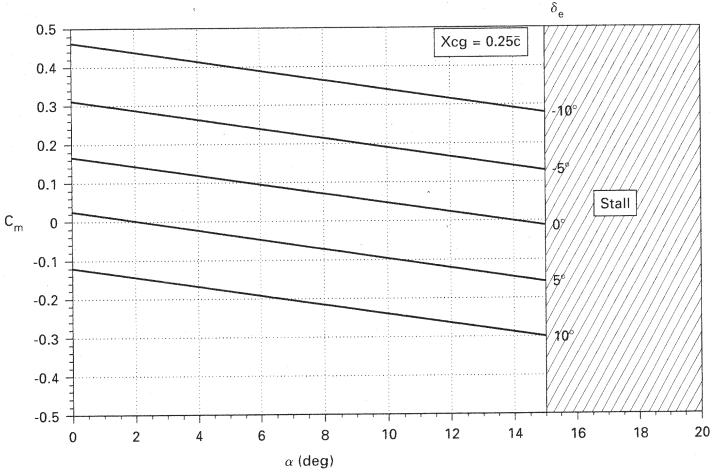
$$\delta_e = -\frac{C_{M0} + C_{M\alpha}\alpha_{eq}}{C_{M\delta_e}}.$$

Solving both these equations simultaneously for δ_e:

$$\begin{aligned}\delta_{e,trim} &= -\frac{C_{M0}C_{L,\alpha} + C_{M,\alpha}C_{L,desired}}{C_{M\delta_e}C_{L,\alpha} - C_{M,\alpha}C_{L,\delta_e}} \\ &= -\frac{C_{M0}C_{L,\alpha} + C_{M,\alpha}C_{L,desired}}{C_{L,\alpha}\tau C_{L\alpha,t}\left(\frac{S_t}{S}\frac{C_{L\alpha,t}}{C_{L\alpha}} - 1\right)V_H}\end{aligned}$$

Reading Data off Plots

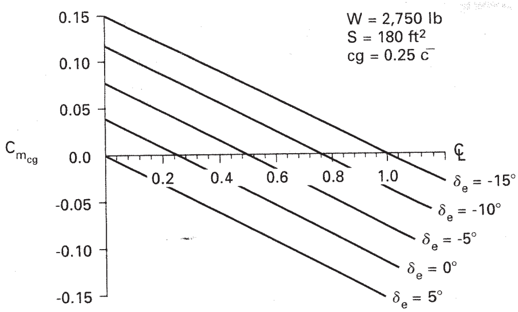
Examples: Moment Curve



We can find $C_{M\alpha} = \frac{\Delta C_M}{\Delta \alpha}$, $C_{M\delta_e} = \frac{\Delta C_M}{\Delta \delta_e}$, and $C_{M0} = C_M$ at $\alpha = 0$.

Reading Data off Plots

Examples: Neutral Point



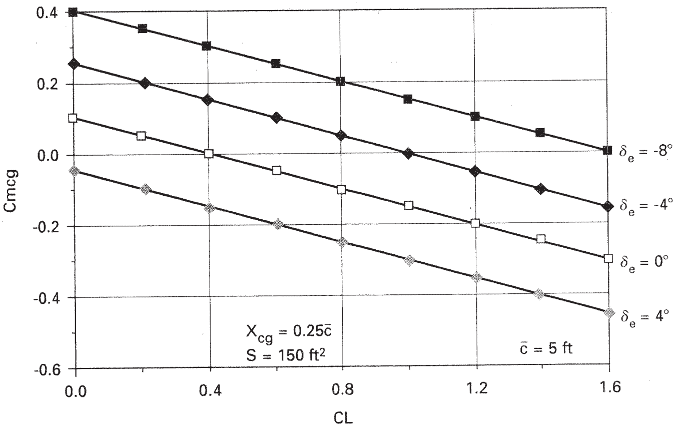
Recall

$$K_n = \frac{X_{NP}}{\bar{c}} - \frac{X_{CG}}{\bar{c}} = \frac{dC_M}{dC_L}$$

This allows us to find the Neutral Point. How to trim for a specific flight condition?

Reading Data off Plots

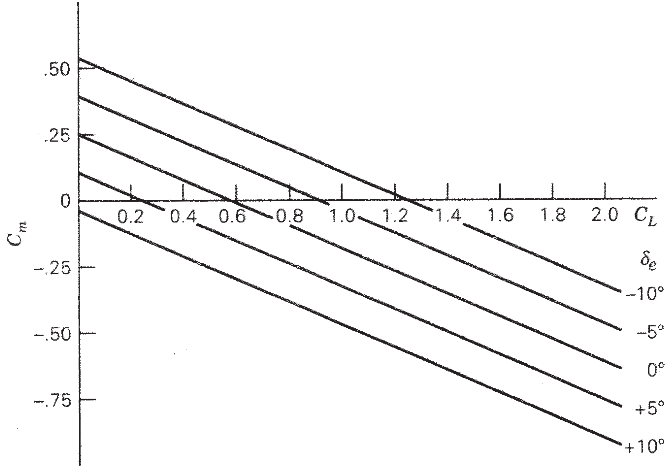
Examples: Neutral Point



To trim for a flight condition, determine the required C_L. Then adjust δ_e until $C_M = 0$ at $C_{L,desired}$.

Reading Data off Plots

Examples: Neutral Point



Conclusion

Today we have covered:

Neutral Point

- Formulae for X_{NP}
- Formulae for Static Margin
- Determining X_{NP} and K_n from plots

Elevators and Trim

- Effect of Elevator on C_M
- Elevator Effectiveness, τ
- Reading Elevator data off of plots

Next Lecture

Next Lecture we will cover:

Numerical Example

- A walk-through of the design process

Directional Stability

- Contributions to Yawing Moment
- Rudder Control

Roll Stability

- Contributions to Rolling Moment
- Aileron Control