Spacecraft and Aircraft Dynamics

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Lecture 7: Example and Directional Control

In this Lecture we will cover:

Numerical Example

- How to design an aircraft for steady-level flight.
- To take into account
  - L=w
  - $\sum M = C_M = 0$
  - $C_{M\alpha} < 0$
  - $C_{M0} > 0$

Directional Stability

- $C_{N\beta} > 0$ means stability.
- Most aircraft need a tail - $C_{N\beta,wf} < 0$.
- How to size a tail so that $C_{N\beta} > 0$.
- The effect of rudder on yawing moment - $C_{N\delta_r}$
- How to estimate aircraft parameters from video.

For a $\frac{1}{25}$ scale model of an airplane, the following geometric data applies

S	$1.50ft^2$ $(.139m^2)$
$\bar{c}$	$6.145in$ $(.1561m)$
l_t	$15.29in$ $(.3884m)$
S_t	$.368ft^2$ $(.0342m^2)$

Aerodynamic Data:

$C_{L\alpha,wf}$	$.077\,\text{deg}^{-1}$	$C_{M0,wf}$	-.018
ε_0	$.72\,\text{deg}$	$C_{L\alpha,t}$	$.064\,\text{deg}^{-1}$
η	1	$\frac{d\varepsilon}{d\alpha}$	.30
ρ	$2.377E-3slug-ft^{-3}$	$\frac{X_{ac}}{\bar{c}}$	.25

Now suppose we have an *Actual Aircraft* of mass $1552.8slugs$ $(22,680kg)$.

1. Find the limits on CG position (X_{CG}) and tail angle measured relative to wing $(i_{tr} = i_{wf} - i_t)$ to ensure static stability and nose-up.
2. For steady-level flight with $\delta_e = 0$, plot the required i_{tr} vs. X_{CG} at $v = 239knots$ $(123m/s)$.

Solution: Note that

$$V_H = \frac{l_t S_t}{\bar{c} S} = \frac{15.29 * .368}{6.145 * 1.5} = .6104$$

$$C_{L\alpha} = C_{L\alpha,wf} + \eta C_{L\alpha,t} \frac{S_t}{S} \left(1 - \frac{d\epsilon}{d\alpha}\right)$$
$$= .077 + 1 * .064 * \frac{.368}{1.5}(1 - .3) = .088 \deg^{-1}$$

We need $C_{M\alpha} < 0$.

$$C_{M\alpha} = C_{L\alpha} \left(\frac{X_{CG}}{\bar{c}} - \frac{X_{AC}}{\bar{c}}\right) - \eta V_H C_{L\alpha,t} \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$
$$= .088 * \left(\frac{X_{CG}}{\bar{c}} - .25\right) - .064 * .6104 * (1 - .3) < 0$$

which means

$$\frac{X_{CG}}{\bar{c}} < .5607$$

Example

Scale Model Airplane

For nose-up flight, we want $C_{M0} > 0$

$$C_{M0} = C_{M0,wf} + \eta C_{L\alpha,t} V_H \left(\varepsilon_0 + i_{wf} - i_t\right) \left[1 - \eta \frac{C_{L\alpha,t}}{C_{L\alpha}} \frac{S_t}{S} \left(1 - \frac{d\varepsilon}{d\alpha}\right)\right]$$

$$= -.018 + 1 * .064 * .6104 * (.72 + i_{tr}) * \left(1 - 1 * \frac{.064}{.088} \frac{.368}{1.5} (1 - .3)\right) > 0$$

or,

$$i_{tr} = i_{wf} - i_t > .193 \, \mathrm{deg}$$

For Part 2, ignoring C_{L0}, we have from $L = W$,

$$\alpha_{eq} = \frac{W}{QSC_{L\alpha}}$$

Now, to go from $1/25$-scale to full scale, we have $m \to 25m$, so

$$S_{full} = 25 * 25 * .139 m^2 = 86.875 m^2$$

Meanwhile,

$$W = 22,680kg * 9.81m/s^2 = 222.491kN,$$

from which we can find the desired α.

$$\alpha = \frac{222491}{\frac{1}{2}1.225 * 123^2 * 86.875 * .088} = 3.141\,\text{deg}$$

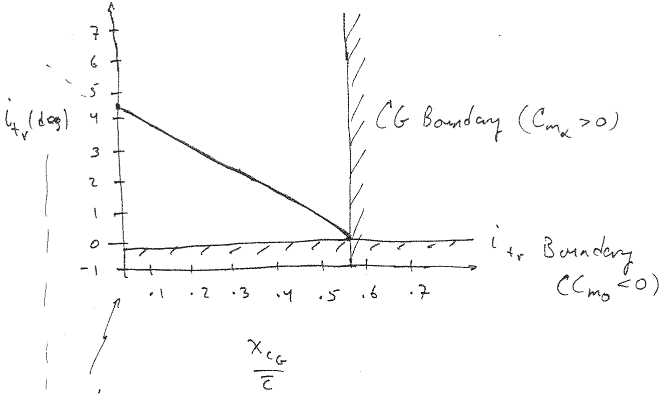
Finally, for moment equilibrium, we need $C_M = C_{M0} + C_{M\alpha}\alpha = 0$.

Plugging in the above values and solving for i_{tr} yields

$$i_{tr} = 4.33 - 8.07\frac{X_{cg}}{\bar{c}}$$

Example

Scale Model Airplane



We can draw a region with three constraints.

- $i_{tr} = 4.33 - 8.07 \frac{X_{cg}}{\bar{c}}$ for steady-level flight.
- $i_{tr} > .193 \deg$ for nose-up.
- $\frac{X_{CG}}{\bar{c}} < .5607$ for stability.

First, let us recall the definition of sideslip angle using free-stream velocity $\vec{V}$.

$$\beta \cong \frac{V_y}{V_x} = \frac{v}{u}$$

Yawing moment is denoted by

$$N = C_N Q b S$$

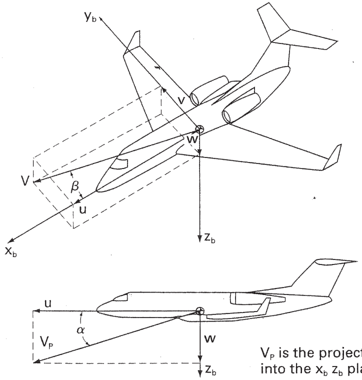
where

- Q is dynamic pressure.
- b is span of the aircraft (tip to tip).
- S is surface area of the wing.

C_N has the form

$$C_N = C_{N0} + C_{N\beta}\beta$$

Typically, $C_{N0} = 0$. Exceptions?



As with longitudinal stability, the equations of motion are easily characterized.

$$\beta_{eq} = -\frac{C_{N0}}{C_{N\beta}}$$

By definition, $+\beta$ is a *negative* yaw rotation.

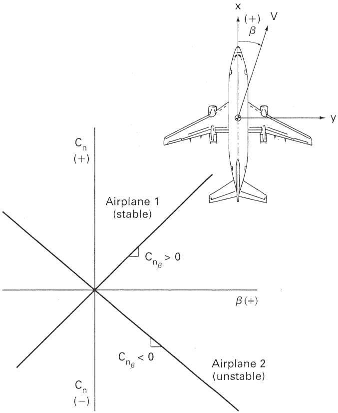
$$\psi(t) = -\beta(t)$$

Thus

$$\frac{I_3}{QSb}\Delta\ddot{\psi}(t) = -C_{N\beta}\Delta\psi(t).$$

This implies

- **Stability** if $C_{N\beta} > 0$.
- **Instability** if $C_{N\beta} < 0$.

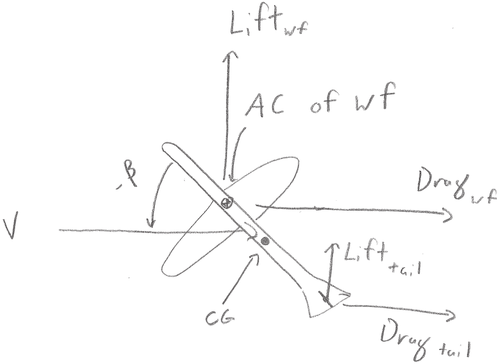


Directional Stability

Moment Contributions

There are two main moment contributions to directional stability

1. wing+fuselage contribution.
2. vertical stabilizer



The **Wing-Fuselage** contribution is difficult to characterize because it is not a ideal lifting surface. However, an approximation can be found as

$$C_{N\beta,wf} = -k_n k_R \frac{S_{fs} l_f}{S_w b}$$

where

- k_n corresponds to geometry.
- k_R corresponds to flight condition.
- S_{fs} is the surface area of the side view of the airplane.
- l_f is the length of the plane

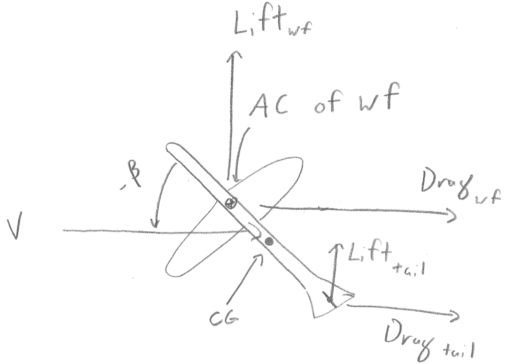
The key point is that all these quantities are typically **positive**. Hence $C_{N\beta,wf} < 0$.

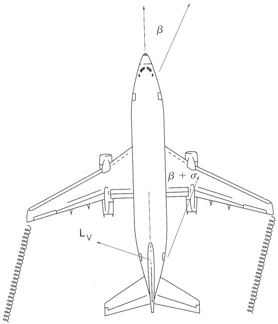
Conclusion: The wing-fuselage by itself is usually **Unstable**.

Why?

Directional Stability

Vertical Stabilizer





σ -Sidewash
due to wing vortices

- The Wing-Fuselage contribution is usually destabilizing because the net force acts to the fore of the CG, creating positive moment.
- To correct, we want to add force aft of the CG.
  - This is done via a lifting surface on the tail.

As with all lifting surfaces, we express the Lift force as

$$Y_v = -C_{L,v} Q_s S_v$$

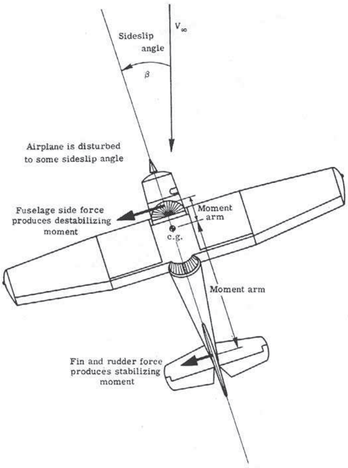
where Q_s and S_v are dynamic pressure at and surface are of the vertical stabilizer.

$$C_{L,v} = C_{L\alpha,v} \alpha_v$$

- α_v is the sideslip angle of the tail, expressed as

$$\alpha_v = \beta + \sigma$$

  - σ is the downwash effect due to the wing



Directional Stability

Moment Coefficient of Vertical Stabilizer

The moment coefficient can now be found as

$$\begin{aligned} C_{n,v} &= \frac{l_v S_v}{Sb} \frac{Q_v}{Q_w} C_{L\alpha,v}(\beta + \sigma) \\ &= V_v \eta_v C_{L\alpha,v}(\beta + \sigma) \end{aligned}$$

where

- l_v is the distance from CG to the Aerodynamic Center of the Vertical Stabilizer.
- $V_v = l_v S_v/(Sb)$ is the volume ratio
- $\eta_v = Q_v/Q_w$ is the efficiency factor.

Again, we assume $\sigma = 0$ at $\beta = 0$ and so we have:

$$\begin{aligned} C_{n,v} &= V_v \eta_v C_{L\alpha,v} \left(1 + \frac{d\sigma}{d\beta}\right) \\ &= C_{n\beta,v} \beta \end{aligned}$$

Directional Stability

Total Moment

Combining the effects of the wing-fuselage and the tail, we have

$$\begin{aligned} C_N &= C_{n0,wf} + (C_{n\beta,v} + C_{n\beta,wf})\beta \\ &= C_{n0,total} + C_{n\beta,total}\beta \end{aligned}$$

where typically $C_{n0,total} = C_{n0,wf} = 0$ and

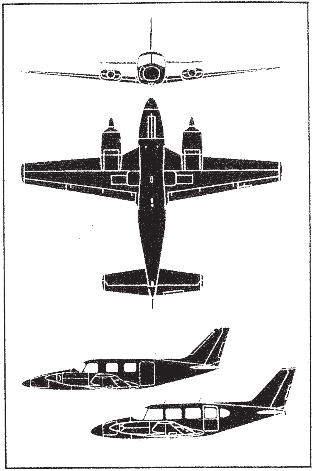
$$C_{n\beta,total} = C_{n\beta,wf} + V_v \eta_v C_{L\alpha,v}\left(1 + \frac{d\sigma}{d\beta}\right)$$

For stability, we design $V_v = l_v S_v/(Sb)$ so that

$$\begin{aligned} V_v &> -\frac{C_{n\beta,wf}}{\eta_v C_{L\alpha,v}\left(1 + \frac{d\sigma}{d\beta}\right)} \\ &= \frac{k_n k_R \frac{S_{fs} l_f}{S_w b}}{\eta_v C_{L\alpha,v}\left(1 + \frac{d\sigma}{d\beta}\right)} \end{aligned}$$

Piper PA-31 Navajo USA

Type: light sports aircraft **Accommodation:** two pilots, four passengers



Dimensions:
Length: 32 ft 7 in (9.9 m)
Wingspan: 40 ft 8 in (12.4 m)
Height: 13 ft (3.9 m)

Weights:
Empty: 3991 lb (1810 kg)
Max T/O: 6500 lb (2948 kg)

Payload: 350 lb (159 kg)

Performance:
Max speed: 261 mph (420 kmh)
Range: 1065 nm (1973 km)
Power plant: two Lycoming TIO-540-A2C piston engines

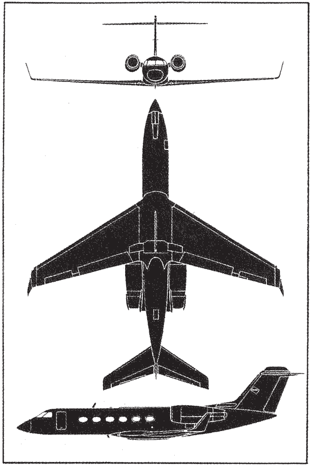
Thrust: 620 hp (231 kW)

Variants:
Navajo C/R; Pressurized Navajo; Navajo Chieftain stretched version

Notes: First flown in 1964, the Navajo has gone through a number of modifications, the Navajo Chieftain being the most recognizable.

Gulfstream III USA

Type: long-range executive transport **Accommodation:** two pilots; optional cabin attendant; 19 passengers



Dimensions:
Length: 83 ft 1 in (25.3 m)
Wingspan: 77 ft 10 in (23.7 m)
Height: 24 ft 4 in (7.4 m)

Weights:
Empty: 38 000 lb (17 236 kg)
Max T/O: 69 700 lb (31 615 kg)

Payload: 1600 lb (726 kg)

Performance:
Max speed: 576 mph (928 kmh)
Range: 4100 nm (7598 km)
Power plant: two Rolls-Royce Spey Mk511-8 turbofans
Thrust: 22 800 lb (101.4 kN)

Variants:
II earlier version; IV advanced version; SRA-4 special missons version; C-20 VIP transport; SMA-3 fisheries protection and SAR aircraft

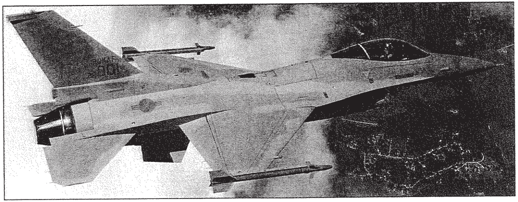
Notes: C-20 used by US armed services and Swedish Air Force (Tp 102). SRA-4s can be fitted with ventral canoe housing ECM, ESM, comms intercept or SLAR. Gulfstream IIs have no winglets, although many refitted with III wing.

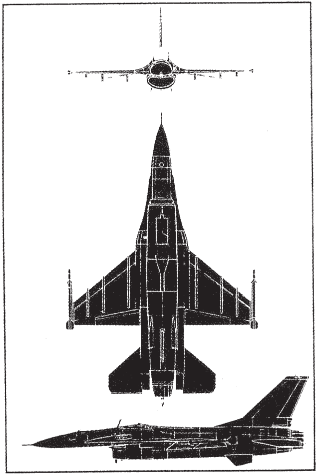
Directional Stability

Example: F/A-16

General Dynamics F-16 Fighting Falcon USA

Type: multi-role fighter **Accommodation:** one pilot





Dimensions:

Length: 49 ft 4 in (15.03 m)
Wingspan: 31 ft (9.45 m)
Height: 16 ft 4 in (5.09 m)

Weights:

Empty: GE - 19 517 lb (8853 kg); PW - 18 726 lb (8494 kg)
Max T/O: 37 500 lb (17 010 kg)

Performance:

Max Speed: above Mach 2
Range: 1480 nm (2642 km)
Powerplant: one General Electric F100-GE-100 or one Pratt & Whitney F100-PW-220 turbofan
Thrust: GE - 29 588 lb (131.6 kN); PW - 29 100 lb (129.4 kN)

Armament:

one 20 mm M61A1 Vulcan cannon; six hardpoints, two wingtip rails; 12 000 lb (5443kg) warload; AIM-120 AMRAAM, AIM-7, AIM-9, Rafael Python 3 AAMs; 30 mm gun pod; AGM-65A; AGM-88 HARM; Harpoon, Penguin; LGBs; bombs; rockets

Variants:

F-16A single-seater; F-16A (ADF); F-16B/D operational trainer; FS-X Japanese licence built derivative

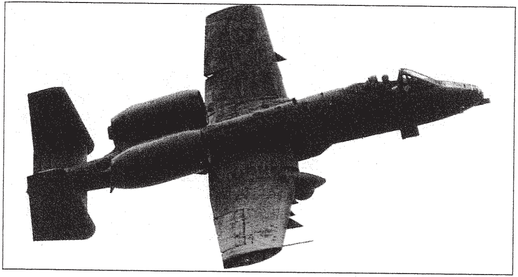
Notes: Specification applies to F-16C. Israeli F-16Ds have a box-like spine fairing housing additional ECM equipment.

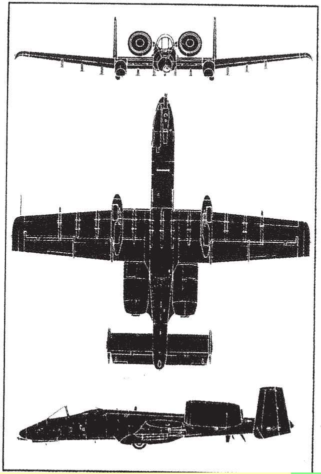
Directional Stability

Example: Flying Car

Fairchild A-10A Thunderbolt USA

Type: close support aircraft **Accommodation:** one pilot





Dimensions:
Length: 53 ft 4 in (16.26 m)
Wingspan: 57 ft 6 in (17.53 m)
Height: 14 ft 8 in (4.47 m)

Weights:
Empty: 23 370 lb (10 710 kg)
Max T/O: 47 400 lb (21 500 kg)

Performance:
Max Speed: 449 mph (722 kmh)
Range: 1080 nm (2000 km)
Powerplant: two General Electric TF34-GE-100 high bypass ratio turbofans
Thrust: 18 130 lb (80.6 kN)

Armament:
one 30 mm GAU-8/A seven-barrelled cannon; 11 hardpoints; 16 000 lb (7257 kg) warload; AGM-65A Maverick; wide range of bombs

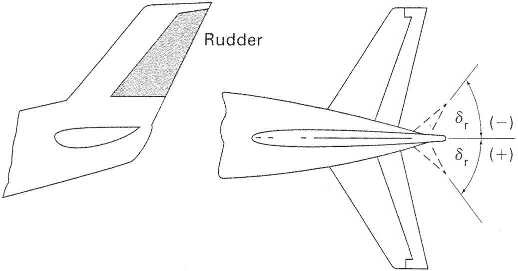
Variants:
OA-10A Fast FAC aircraft

Notes: The pilot is protected by a titanium `bathtub' capable of withstanding 23 mm gun fire.

Directional Control

Rudder Moment

The rudder is a *control surface* attached to the vertical tail.



A **Positive** deflection produces a **Negative** Yawing moment.

$$N_r = -l_v Y_r$$

where

$$Y_r = Q_v S_v \frac{dC_{L,v}}{d\alpha} \frac{d\alpha}{d\delta_r} \delta_r$$

Directional Control

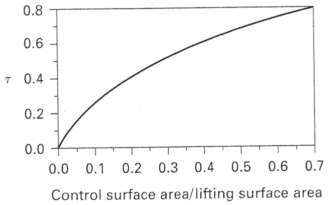
Rudder Moment

Define $C_{n,r} := \frac{N_r}{QSb}$. Then as for all lifting surfaces:

$$\begin{aligned} C_{n,r} &= -\frac{Q_v}{Q}\frac{l_v S_v}{Sb} C_{L\alpha,v}\frac{d\alpha}{d\delta_r}\delta_r \\ &= -\eta_v V_v C_{L\alpha,v}\tau\delta_r \\ &= C_{n,\delta_r}\delta_r \end{aligned}$$

where τ is as defined for elevator deflection

$$C_{n,\delta_r} = -\eta_v V_v C_{L\alpha,v}\tau$$



Directional Control

Example: Sideslip Landing

Question: In the movie, a rudder deflection of $\cong 30\,\deg$ produces a stable sideslip angle of $\cong 50\,\deg$. Estimate $C_{n\beta,wf}$. Neglect Downwash, use $\eta_v = 1$, $C_{L\alpha,v} = .1\,\deg^{-1}$, $\frac{S_r}{S_t} = .6$ and $V_v = .8$.

Answer: The moment equation is given by

$$C_N = C_{N\beta}\beta + C_{N\delta_r}\delta_r$$

where

$$C_{N\delta_r} = -\eta_v V_v C_{L\alpha,v}\tau,$$

and

$$C_{N\beta} = C_{n\beta,wf} + V_v C_{L\alpha,v}$$

Now, fixing δ_r, the equilibrium is when $C_N = 0$, so

$$\frac{\beta_{eq}}{\delta_r} = -\frac{C_{N\beta}}{C_{N\delta_r}} = \frac{C_{n\beta,wf} + V_v C_{L\alpha,v}}{V_v C_{L\alpha,v}\tau}$$

Directional Control

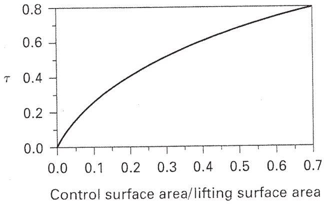
Example: Sideslip Landing

- In the movie, a negative rudder deflection caused a negative $\beta_e q$. This implies $C_{N\beta} > 0$ since $C_{N\delta_r}$ is always negative. Thus the aircraft is stable.
- Notice that since $\beta_{eq} > \delta_r$, $C_{n\beta,wf}$ must be almost positive, or almost stable without the tail.

Continuing, we solve for

$$C_{n\beta,wf} = \left(\frac{\beta_{eq}}{\delta_r}\tau - 1\right) V_v C_{L\alpha,v}$$

Thus the wing-fuselage is stable if $\beta_{eq}\tau > \delta_r$. In this case, have $\frac{S_r}{S_t} = .6$ implies $\tau \cong .7$ from the plot



Finally, we conclude

$$\beta_{eq}\tau = 50 * .7 = 35\,\deg \qquad \text{and} \qquad \delta_r = 30\,\deg$$

Hence, we have the wing-fuselage is stable. Additionally, estimate $V_H = .8$ and $C_{L\alpha,v} = .1$. Then

$$C_{n\beta,wf} = .17 * .8 * .1 = +.013\,\deg^{-1}$$

Conclusion

In this Lecture, you learned:

The Design Process

- How to design an aircraft for steady-level flight.
- To take into account
  - L=w
  - $\sum M = C_M = 0$
  - $C_{M\alpha} < 0$
  - $C_{M0} > 0$

Directional Stability

- $C_{N\beta} > 0$ means stability.
- Most aircraft need a tail - $C_{N\beta,wf} < 0$.
- How to size a tail so that $C_{N\beta} > 0$.
- The effect of rudder on yawing moment - $C_{N\delta_r}$
- How to estimate aircraft parameters from video.

Next Lecture: Axial Stability and beyond

We will wrap up static stability with a discussion of axial stability

- Fuselage Contribution
- Dihedral Effect
- Aileron Control

We will then move on to

- 3-D stability
- Equations of motion in a rotating reference frame.