Spacecraft and Aircraft Dynamics

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Lecture 12: Interplanetary Mission Planning

Introduction

In this Lecture, you will learn:

Sphere of Influence

- Definition

Escape and Re-insertion

- The light and dark of the Oberth Effect

Patched Conics

- Heliocentric Hohmann

Planetary Flyby

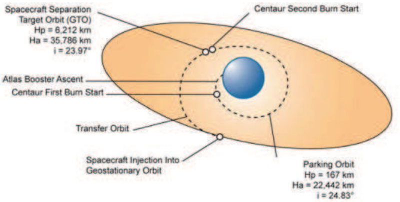
- The Gravity Assist

The Sphere of Influence Model

Three-Body Motion

Consider a Simple Earth-Moon Trajectory.

1. Launch
2. Establish Parking Orbit
3. Escape Trajectory
4. Arrive at Destination
5. Circularize or Depart Destination



The big difference is that now there are 3 bodies.

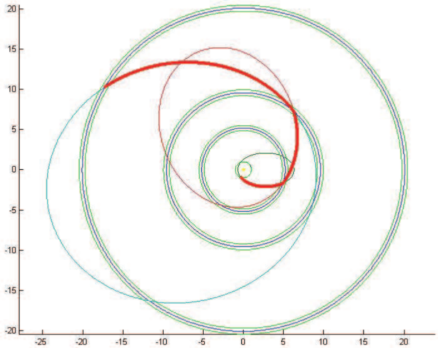
- We only know how to solve the 2-body problem.
- Solving the 3-body problem is beyond us.

Patched Conics

For interplanetary travel, the problem is even more complicated.

Consider the Figure

- The motion is elliptic about the sun.
- The motion is affected by the planets
  - Interference only occurs in the green bands.
  - Motion about planets is hyperbolic.



The solution is to break the mission into segments.

- During each segment we use *two-body motion*.
- The third body is a disturbance.

Sphere of influence

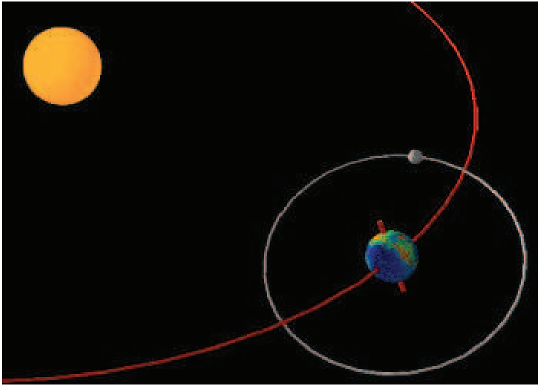
The Wrong Definition

Question: Who is in charge??

- The Sphere of Influence of A stops when A is no longer the dominant force.
- What do we mean by dominant?

Wrong Definition:
The Sphere of Influence of A is the region A exerts the largest gravitational force.

This would imply the moon is not in earth's Sphere of Influence!!!



Sphere of influence

The Sun's Perspective

Sun Perspective: Lets group the forces as central and disturbing. Consider motion of a spacecraft relative to the sun:

$$\ddot{\vec{r}}_{sv} + \frac{G(m_s + m_v)}{\|\vec{r}_{sv}\|^3}\vec{r}_{sv} = -Gm_p\left[\frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3} + \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3}\right]$$

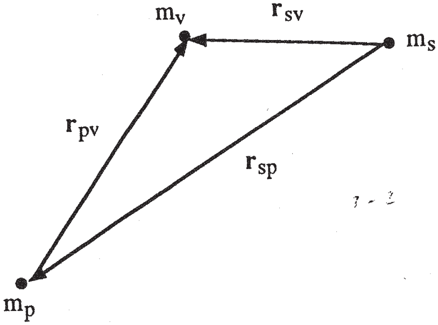
where p denotes planet, v denotes vehicles and s denotes sun.

The Central Force is

$$\vec{F}_{central,s} = \frac{G(m_s + m_v)}{\|\vec{r}_{sv}\|^3}\vec{r}_{sv}$$

The Disturbing Force is

$$\vec{F}_{dist,s} = -Gm_p\left[\frac{\vec{r}_{pv}}{\|\vec{r}_{pv}\|^3} + \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3}\right]$$



Planet Perspective: The motion of the spacecraft relative to the planet is

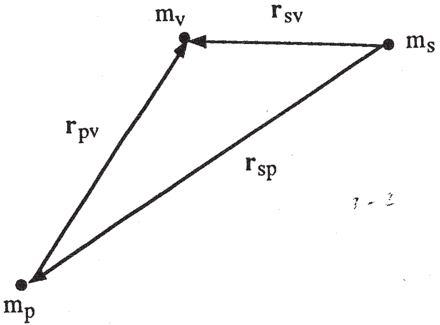
$$\ddot{\vec{r}}_{pv} + \frac{G(m_p + m_v)}{\|\vec{r}_{pv}\|^3}\vec{r}_{pv} = -Gm_s\left[\frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3} + \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3}\right]$$

The Central Force for the planet is

$$\vec{F}_{central,p} = \frac{G(m_p + m_v)}{\|\vec{r}_{pv}\|^3}\vec{r}_{pv}$$

The Disturbing Force for the planet is

$$\vec{F}_{dist,p} = -Gm_s\left[\frac{\vec{r}_{sv}}{\|\vec{r}_{sv}\|^3} + \frac{\vec{r}_{sp}}{\|\vec{r}_{sp}\|^3}\right]$$



Definition 1.

An object is in the **Sphere of Influence**(SOI) of body 1 if

$$\frac{\|\vec{F}_{dist,1}\|}{\|\vec{F}_{central,1}\|} < \frac{\|\vec{F}_{dist,2}\|}{\|\vec{F}_{central,2}\|}$$

for any other body 2.

That is, the ratio of disturbing force to central force determines which planet is in control.

For planets, an approximation for determining the SOI of a planet of mass m_p at distance d_p from the sun is

$$R_{SOI} \cong \left(\frac{m_p}{m_s}\right)^{2/5} d_p$$

Sphere of influence

Table 7.1 Sphere of Influence Radii

Celestial Body	Equatorial Radius (*km*)	SOI Radius (*km*)	SOI Radius (*body radii*)
Mercury	2487	1.13×10^5	45
Venus	6187	6.17×10^5	100
Earth	6378	9.24×10^5	145
Mars	3380	5.74×10^5	170
Jupiter	71370	4.83×10^7	677
Neptune	22320	8.67×10^7	3886
Moon	1738	6.61×10^4	38

Example: Lunar Lander

Problem: Suppose we want to plan a lunar-lander mission. Determine the spheres of influence to consider for a patched-conic approach.

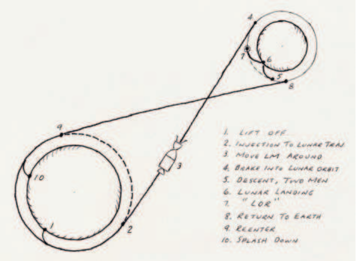
- The moon orbits at a distance of 384,000km.
- The Sphere of influence of the earth is of radius 924,000km.
- The sphere of influence of the moon is of radius 66,100km.

Solution: The spacecraft will transition to the lunar sphere at distance

$$r = 384,000 - 66,100 = 317,900 km$$

Thus we will need a plane change. A reasonable mission design is

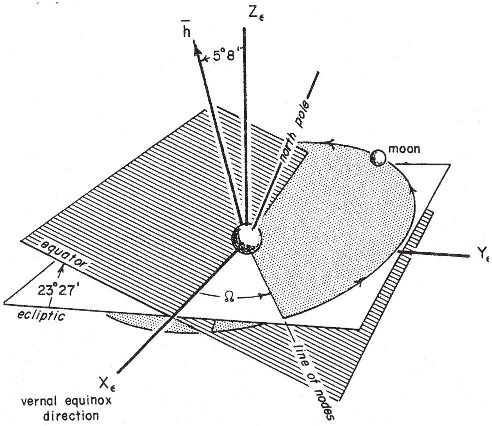
1. Depart earth on a Hohmann transfer to radius $317,900$ km.
2. Perform inclination change near apogee.
3. Enter sphere of influence of the moon.
4. Establish parking orbit.



Example: Lunar Lander

Additionally, a Plane Change is needed.

- Note that the lunar orbit is inclined at about 5.8° to the ecliptic plane.
- The inclination of the lunar orbit is almost fixed with respect to the ecliptic.
- Not fixed but not the equatorial plane.
- Inclination to equator varies by $21.3^\circ \pm 5.8^\circ$ every 18 years.



5 Stage Lunar Intercept Mission

First Stage Lunar Tug Assist

Every mission is different.

- It is impossible to cover every scenario

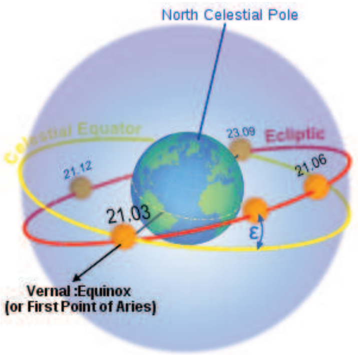
Instead, Let's go through an example.

- Can serve as a template.

Problem: Design an Earth-Venus rendez-vous. Final Venus orbit should be posigrade of altitude 500km.

Solution: We begin in an initial parking orbit.

- Orbital plane aligned with ecliptic plane
  - $i \cong 23^\circ$
- Circular orbit.
  - Radius $r \cong 6578km$



Interplanetary Hohmann Transfer

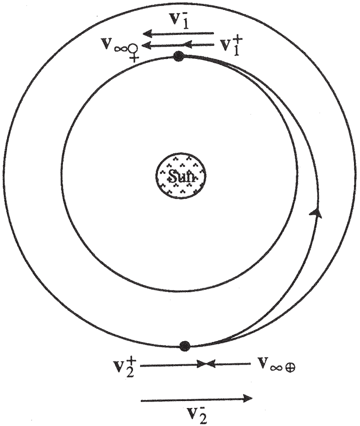
Design a Hohmann transfer from Earth to Venus.

Naturally, the perigee and apogee velocities of the transfer ellipse are

$$v_p = \sqrt{2\mu_{sun}\frac{r_e}{r_v(r_e + r_v)}}$$

$$v_a = \sqrt{2\mu_{sun}\frac{r_v}{r_e(r_e + r_v)}}$$

Note that because Venus is an inner planet, apogee velocity occurs at Earth



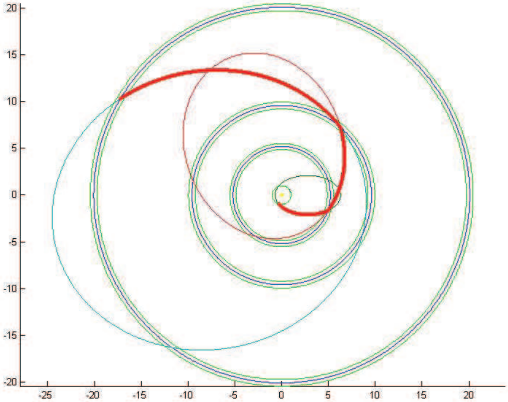
The Hohmann transfer is defined using the Sphere of Influence of the *Sun*

- Velocities are in the **Heliocentric Frame**!

Interplanetary Hohmann Transfer

We can use the Hohmann transfer because the voyage will take place exclusively in the sun's frame of reference.

- The earth orbits at radius $1au = 1.5 \cdot 10^8 km = 23,518ER$.
- The SOI of the earth is only $145ER$, or $.5\%$.



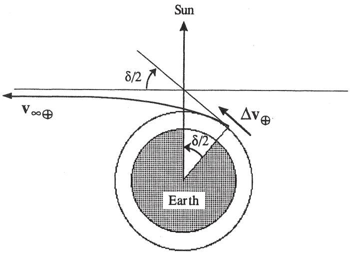
Interplanetary Hohmann Transfer

Injection (v_a)

Problem: How to achieve the initial v_a?

The initial velocities v_a and v_p are in the *Heliocentric* frame.

- To achieve v_a requires an initial Δv
- Initial Δv will be in the *Geocentric* frame.
  - Preferably in low orbit (Oblerth Effect)



In the Geocentric Frame, we require

$$v_\infty + v_e = v_a$$

v_e is velocity of the earth in heliocentric frame. Thus the expression for v_∞ is

$$v_\infty = \sqrt{-\frac{\mu}{E}} = \sqrt{v_f^2 - \frac{2\mu}{r_{park}}}$$

where v_f is the speed at injection and r_{park} is the parking radius.

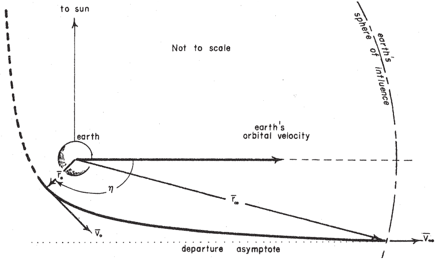
$$\mu_{sun} = 1.327 \cdot 10^{11}, a_{earth} = 1.49 \cdot 10^8$$

It is now easy to compute

$$v_\infty = v_p - v_e = 27.34 - 29.84 = -2.48 km/s$$

We can now solve for v_f.

$$v_f = \sqrt{(v_p - v_e)^2 + \frac{2\mu}{r_{park}}}$$



To calculate the initial Δv, use $v_i = \sqrt{\mu / r_{park}}$ for velocity of the parking orbit.

$$\Delta v_1 = v_f - v_i = 11.28 km/s - 7.78 km/s = 3.5 km/s$$

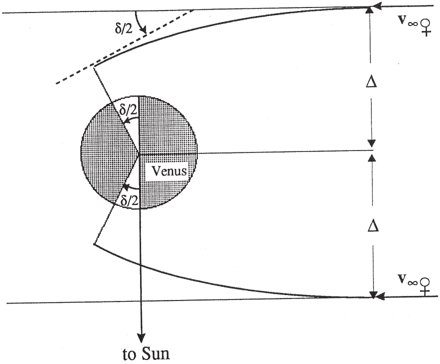
Arrival at Venus

$$R_v = 6187km, \quad \mu_v = 324859, \quad a_{venus} = 1.08 \cdot 10^8$$

Our incoming velocity in the Venus-frame is

$$v_{\infty,v} = v_p - v_v = 37.81km/s - 35.09km/s = 2.71km/s$$

Because the velocity is positive, we will enter from the back door.



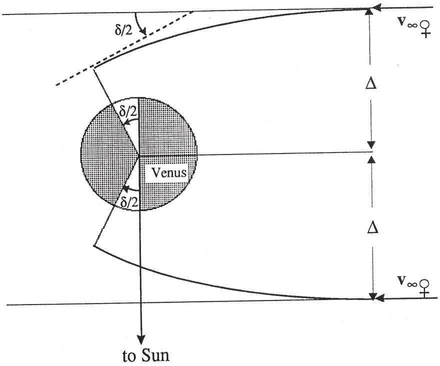
Arrival at Venus

For orbital insertion, we want to perform a retrograde burn at periapse.

- We need our periapse to be $r_{des} = 6687km$.
- The a of the injected orbit is

$$-\frac{\mu_v}{2a} = E = \frac{1}{2}v_{\text{inf},v}^2$$

- a cannot be modified.



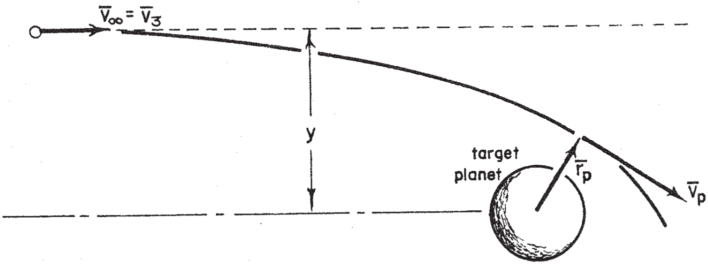
- We calculate $a = -\mu_v/v_{inf,v}^2 = -44,232$.
- To achieve $r_p = a(1-e)$, we need

$$e = 1 - \frac{r_p}{a} = 1.15$$

Arrival at Venus

To achieve the desired $e = 1.15$, we control the conditions at the *Patch Point*.

- We do through the angular momentum, h.



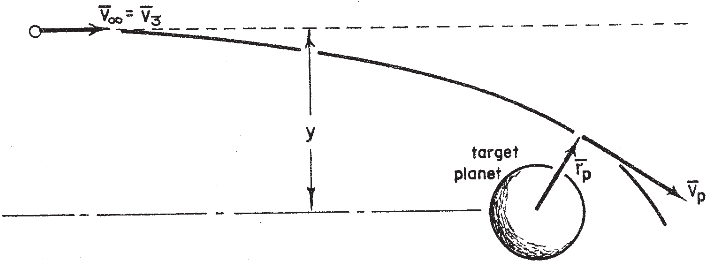
We can control the **Target Radius**, Δ through small adjustments far from the planet. Angular momentum can be controlled exactly through target radius, Δ.

$$h_v = \Delta v_{\infty,v}$$

Arrival at Venus

Recall that p is defined only by angular momentum

$$p = \frac{h^2}{\mu} = \frac{\Delta^2 v_{\infty,v}^2}{\mu_v}$$



Since

$$p = a(1-e^2)$$

and a is fixed, we can solve for Δ,

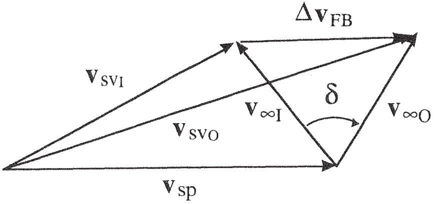
$$\Delta = \sqrt{\frac{p\mu_v}{v_{\infty,v}^2}} = \sqrt{\frac{a(1-e^2)\mu_v}{v_{\infty,v}^2}} = 25,120km$$

Messenger Probe to Mercury

Gravity Assist Trajectories

The same approach can be used to design gravity assist trajectories. In 2-dimensions, this is

$$\vec{v}_f = R_1(\delta)\left(\vec{v}_i - v_{planet}\right) + \vec{v}_{planet}$$



Example: If $\delta = 180°$ and $\vec{v}_i = \begin{bmatrix} -2 \\ 0 \end{bmatrix} km/s$ and $\vec{v}_p = \begin{bmatrix} 2 \\ 0 \end{bmatrix} km/s$, then

$$v_f = R(180°)\begin{bmatrix} -4 \\ 0 \end{bmatrix} km/s + \begin{bmatrix} 2 \\ 0 \end{bmatrix} km/s = \begin{bmatrix} 4 \\ 0 \end{bmatrix} km/s + \begin{bmatrix} 2 \\ 0 \end{bmatrix} km/s = \begin{bmatrix} 6 \\ 0 \end{bmatrix} km/s$$

Thus the probe was able to *triple* its velocity!

Gravity Assist Trajectories

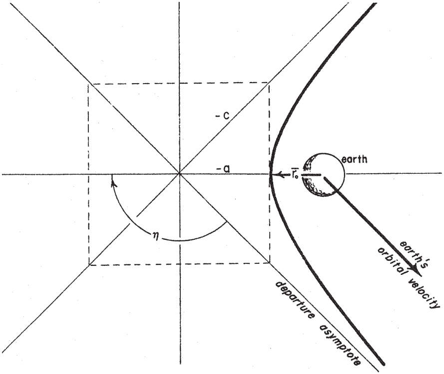
To achieve the desired turning angle, we must control the geometry

The turning angle δ is given by

$$2\cos^{-1}\frac{1}{e}$$

Recall

$$a = -\mu_{planet}/\|\vec{v}_i - \vec{v}_{planet}\|^2$$



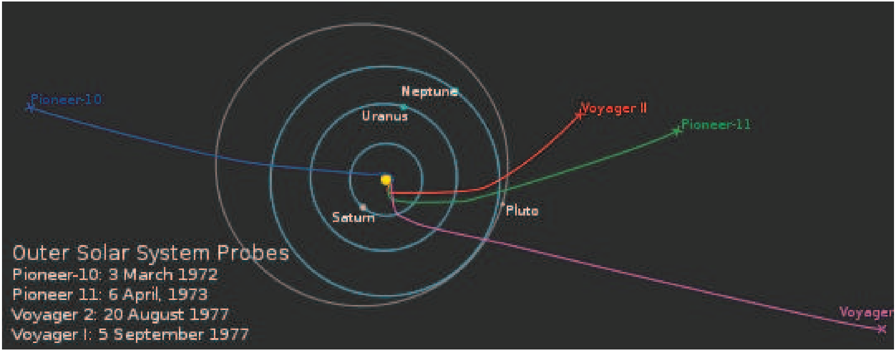
Then the eccentricity can be fixed by the target radius as

$$\Delta = \sqrt{\frac{a(1-e^2)\mu_{planet}}{\|\vec{v}_i - \vec{v}_{planet}\|^2}} = 25,120km$$

In 3 dimensions, the calculations are more complex.

Trajectories for Voyager 1 and Voyager 2 Spacecraft

Trajectories for Voyager 1, Voyager 2, and Pioneer Spacecraft



Summary

This Lecture you have learned:

SPACECRAFT DYNAMICS

Next Lecture: Final Exam.