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Stability and computation of roots in delayed systems of neutral type

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Abstract

In this paper we give methods for checking the location of poles of neutral systems with multiple delays. These are of use in determining exponential stability and H_∞-stability in the single delay case.

Keywords

SOS tools, delay system, neutral system

20.1 Statement of the problem

We consider the problem of stability of systems with characteristic equations of the form

$$G(s) = G_1(s) + \sum_{i=2}^{n} G_i(s) e^{-\tau_i s}, \qquad \text{where} \qquad G_i(s) = \sum_{j=1}^{m} a_{ij} s^j,$$

for $a_{ij} \in \mathbb{R}$ and $\tau_i \geq 0$. Suppose we are given the values of a_{ij} and would like to determine whether the system is stable, either in the exponential or H_∞ sense, for a given set of values of τ. In this paper, we give results which allow us to answer two distinct questions.

1. **Delay-Independent Stability:** Is G exponentially stable for $\tau_i \geq 0$?
2. **Delay-Dependent Stability:** For given h_i, is G H_∞-stable for $\tau_i \in [0, h_i]$?

Our work gives results which allow us to reformulate the problem in terms of semialgebraic sets. We then use Positivstellensatz results to express the problem as convex optimization over sum-of-squares polynomials. We use semidefinite programming to solve the optimization numerically. We use the version of the Positivstellensatz given by Stengle [3].

Theorem 20.1.1 (Stengle). *The following are equivalent*

1. $\left\{x : \begin{matrix} p_i(x) \geq 0 & i = 1, \ldots, k \\ q_j(x) = 0 & j = 1, \ldots, m \end{matrix}\right\} = \emptyset$
2. *There exist* $t_i \in \mathbb{R}[x]$, $s_i, r_{ij}, \ldots \in \Sigma_s$ *such that*

$$-1 = \sum_{i=1}^{m} q_i t_i + s_0 + \sum_{i=1}^{k} s_i p_i + \sum_{\substack{i,j=1 \\ i \neq j}}^{k} r_{ij} p_i p_j + \cdots$$

Here $\mathbb{R}[x]$ denotes the set of real-valued polynomials in variables x and Σ_s denotes the subset of $\mathbb{R}[x]$ which admit a sum-of-squares representation. For a given degree bound, the conditions associated with Stengle's positivstellensatz can be represented by a semidefinite program since for any $s_i \in \Sigma_s$, there exists a matrix $Q \geq 0$ such that $s(x) = Z(x)^T Q Z(x)$, where Z is a vector of monomials in x. The connection between semidefinite programming and sum-of-squares was first made by Parillo [1].

Delay-Independant Stability In this case, we use the following very simple stability condition.

Proposition 20.1.2. *Suppose that for some* $\epsilon > 0$, $\{s : G_1(s) + \sum_{i=2}^{n} G_i(s) z_i = 0, \operatorname{Re} s \geq -\epsilon, \|z_i\|^2 \leq 1 + \epsilon\} = \emptyset$. *Then* G *is exponentially stable for any* $\tau_i \geq 0$.

Using the Positivstellensatz, we construct a semidefinite program which checks the conditions of the Lemma. This is illustrated using a number of numerical examples.

Robust Delay-Dependent Stability In this case, we use an approach first considered by Zhang et al. [4]. This method was based on two principles; 1) The location of the rightmost root of G is a continuous function of the values of the delay τ and 2) A robust version of the Padé approximation can be used to enclose the function $e^{-j\omega}$ on the imaginary axis.

For neutral systems, principle 1 holds for $\tau > 0$, but not necessarily at $\tau = 0$. Therefore, we must check that new roots appear in the left half-plane for infinitessimal τ and in the particular case of a single delay, we have a condition [2] which characterizes this. In the case of multiple commensurate delays, we use a more conservative condition given in terms of the $a_{i,n}$.

Once the above conditions have been satisfied, we can apply robust Padé approximants in the spirit of [4]. We can then use the Positivstellensatz to construct semidefinite programming conditions. This approach is illustrated with numerical examples.

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