Arizona State University
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Optimal Thermostat Programming and Electricity Prices for Customers with Demand Charges

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Power Companies Pay For Fuel & Generators

A simplified model for cost of generating electricity is a combination of

1. **Cost of fuel** required to generate the total energy (kWh) consumed by users

   A common model is: $$\boxed{\text{cost of fuel} = a \int q(t)dt}$$

   $q(t)$ (kW): power consumed by users, a (\$/kWh): cost of fuel required to produce the next kWh

2. **Cost of building & maintaining generators** to accommodate for the maximum total power (kW) consumed by users

   A simple model can be:

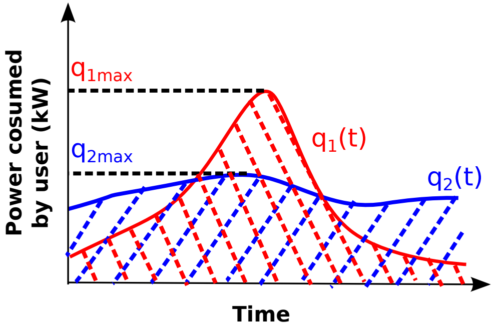
   $$\boxed{\text{Cost of building \& maintaining generators} = b \sup_{t \in \text{on-peak}} q(t)}$$

   b (\$/kW): cost of installing the next kW of generating capacity

Current Pricing Strategies Do Not Charge For Max Power

- Most power companies use **flat** or **Time-of-Use** (ToU) pricing

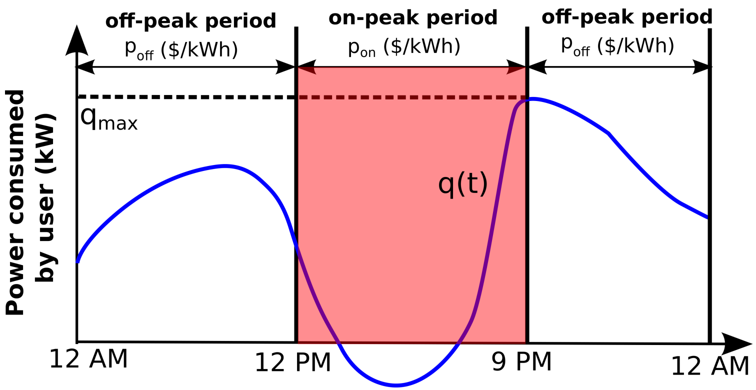
↳ **Flat pricing:** Charges are independent of when energy is used



$$\int q_1(t)dt \times \frac{\text{price}}{kWh} = \int q_2(t)dt \times \frac{\text{price}}{kWh}$$

Electricity bills independent of $q_{1\,\max}$ & $q_{2\,\max}$

↳ **ToU pricing**: Does not explicitly charge for max power used



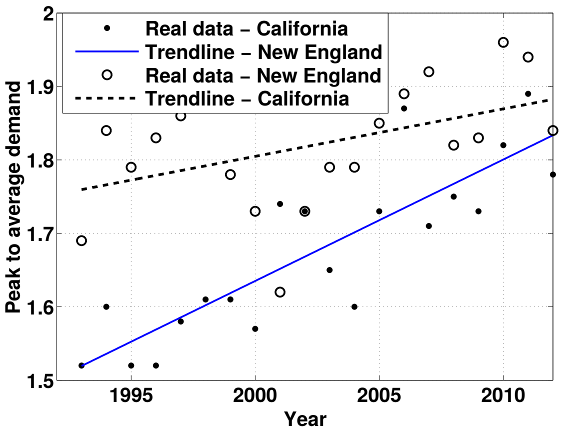
$$\text{Elect. Bill} = p_{\text{off}} \int_{off-peak} q(t)dt + p_{\text{on}} \int_{on-peak} q(t)dt$$

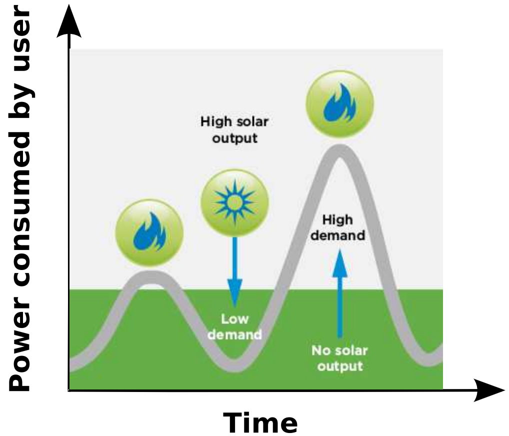
Large peak does not necessarily result in a large monthly bill

Current Pricing Strategies Are Problematic For Power Companies

- **Fact 1:** The ratio of **maximum power** used per year to **average power** used per year is setting records in the US!

  ↳Partially due to increasing **integration of renewables**, e.g., solar



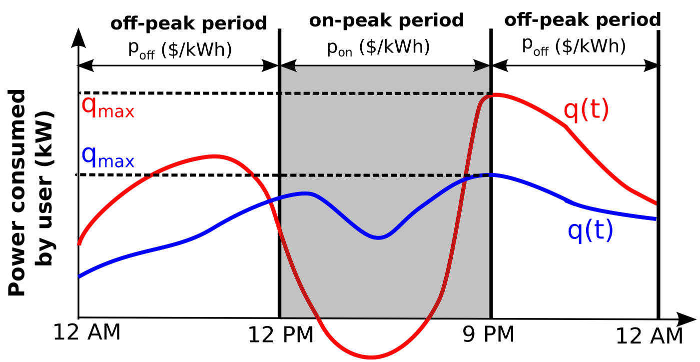


- **Fact 2:** Integration of renewables **does NOT** affect maximum power consumption, but reduces the total power sold by power companies ⇒ **revenue decreases**

- **Consequence:** Power companies won't have enough revenue to supply for electricity without raising the prices

Demand Charge: A Solution To The Revenue Problem

- **Demand charge**: A monthly charge proportional to the maximum power consumed by the user during the on-peak hours of a month
- A combination of off-peak, on-peak and demand charges can differentiate between "**good**" and "**bad**" user behavior



Electricity Bill $=$

$$\underbrace{p_{\text{off}} \int_{t \in \text{off-peak}} q(t)dt}_{\text{on-peak period charge}} + \underbrace{p_{\text{on}} \int_{t \in \text{on-peak}} q(t)dt}_{\text{off-peak period charge}} + \underbrace{p_d \sup_{t \in \text{on-peak}} q(t)}_{\text{demand charge}}$$

How Can Power Companies Optimize Their Prices?

Power companies can solve the following optimization problem:

- **Objective:** minimize the cost of generating electricity

$$\min_{p_{\text{on}}, p_{\text{off}}, p_d} \left(\underbrace{a \int_{t=0}^{t=24} g(t) dt}_{\text{fuel cost}} + \underbrace{b \sup_{t \in \text{ on-peak period}} g(t)}_{\text{cost of building generators}} \right)$$

  - $g(t)$: power (kW) generated at time t
  - a (\$/kWh): cost of fuel required to produce the next kWh
  - b (\$/kW): cost of installing the next kW of production capacity

- **Constraint:**
  - Equality of **generation** $(g(t))$ and **power** $(q_{\text{user}}(t))$ consumed by users:

$$\boxed{g(t) = q_{\text{user}}(t, p_{\text{off}}, p_{\text{on}}, p_d) \quad \forall t}$$

- **Variables:** on-peak, off-peak and demand prices: $p_{\text{on}}, p_{\text{off}}, p_d$

Power Companies Need A Model For User Behavior

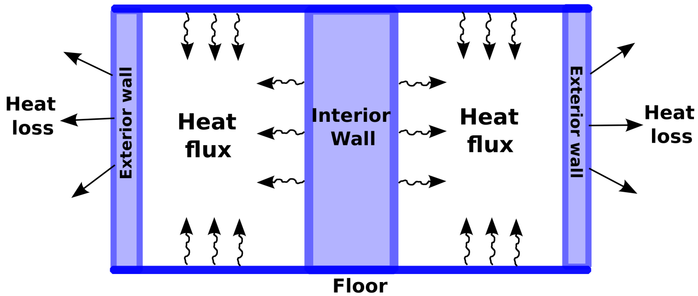
- To optimize electricity prices, we need a **model** for **users' power consumption** which;
  1. Predicts how much electricity would a rational user consume, given the prices

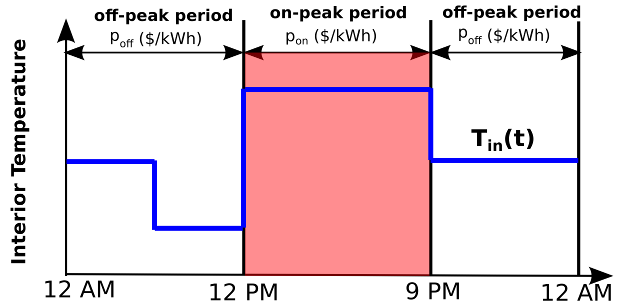
- **Question**: How can a rational user reduce his electricity bill?
  - One way is to reduce HVAC load by using **Energy storage**
    1. Energy storage in residential batteries allows users to shift peaks from high-demand hours to another hours
    2. Using walls/floors as **thermal energy storage**: A **free alternative** to batteries

Precooling: As Time-of-Use Strategy To Reduce Bills

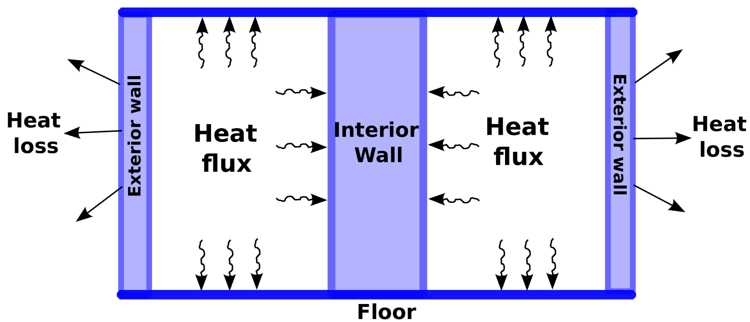
Precooling exploits thermal energy storage in walls to shift loads:

- Cool down walls/floors when electricity is cheap



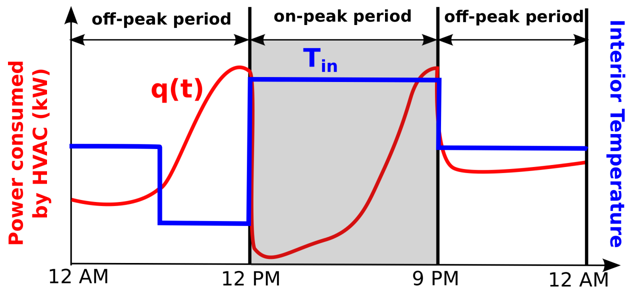


- Cold walls will reduce the load on HVAC during on-peak hours - thus reducing the electricity bill



Precooling Fails When Demand Charges Are Applied

- Precooling does **NOT** reduce max power consumption. **Why?**
  1. Thermal storage in the walls **depletes** before the end of the on-peak period
  2. Then HVAC will remain as the **only** cooling mechanism
  3. At the end of on-peak period, same load will be on HVAC as if no precooling had occurred



- When demand charges exist, thermostat programming is difficult!

  ↳ Thermal storage is governed by the heat equation - A PDE

  ↳ Heat equation inherently has **latency**, thus a good strategy may involve counter-intuitive temperature settings

How Do Thermostat Settings Affect Energy Consumption?

Power consumed by user is a combination of heat **loss to outside** and heat **given to/taken from interior walls**

$$\boxed{q_{\text{user}}(t) = q_{\text{loss}}(t) + q_{\text{wall}}(t) \quad \forall k}$$

- Heat loss $q_{\text{loss}}(t)$ is modeled by a **linear heat sink** and can be controlled by interior temperature T_{in}:

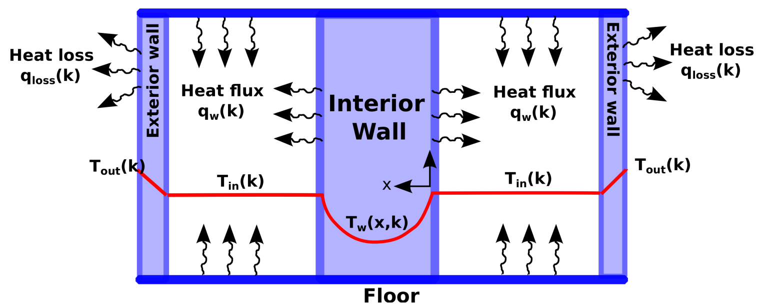
$$q_{\text{loss}}(t) = \frac{T_{\text{out}}(t) - T_{\text{in}}(t)}{R_w}$$

T_{out} : Outside temperature R_w: thermal resistance

- Heat thru walls $q_{\text{wall}}(k)$ is modeled by the **Heat equation** (PDE):

$$\frac{\partial T_w(t,x)}{\partial t} = \alpha \frac{\partial^2 T_w(t,x)}{\partial x^2}$$

$$q_{\text{wall}}(k) = 2C_w \frac{\partial T_w}{\partial x}(t,0)$$



How Do Rational Users Minimize Their Bill?

User can solve a discrete-time thermostat programming problem with

- **Objective:** minimize the electricity bill

$$\min_{T_{\text{in}}(k)} \left(\underbrace{30\, p_{\text{off}} \sum_{k \in I_{\text{off}}} q_{\text{user}}(k)}_{\text{OFF-peak period charge}} + \underbrace{30\, p_{\text{on}} \sum_{k \in I_{\text{on}}} q_{\text{user}}(k)}_{\text{ON-peak period charge}} + \underbrace{p_d \sup_{k \in I_{\text{on}}} q_{\text{user}}(k)}_{\text{demand charge}} \right)$$

- **Constraints:**
  1. Interior temperature with a certain bound:

$$T_{\min} \le T_{\text{in}}(k) \le T_{\max} \quad \forall k$$

  2. Energy conservation:

$$q_{\text{user}}(k) = q_{\text{loss}}(T_{\text{in}}(k), T_e(k)) + q_{\text{wall}}(T_w(x,k)) \quad \forall k$$

  3. Discretized heat dynamics: $T_w(k+1) = A\, T_w(k) + B\, T_{\text{in}}(k)$

- **Variables:** Interior temperature $T_{\text{in}}(k)$ over time

A Reformulation of User's Problem Can Be Solved By Dynamic Programming

- We reformulate the user's problem

$$\begin{array}{ll} \min\limits_{T_{\text{in}}(k)} & 30\, p_{\text{off}} \sum\limits_{k \in I_{\text{off}}} q(k) + 30\, p_{\text{on}} \sum\limits_{k \in I_{\text{on}}} q(k) + \boxed{p_d \sup\limits_{k \in I_{\text{on}}} q(k)} \\ \text{subject to} & q(k) = q_{\text{loss}}(T_{\text{in}}, T_{\text{out}}) + q_{\text{w}}(T_w) \quad \forall k \\ & T_w(k+1) = f(T_w(k), T_{\text{in}}) \quad \forall k \\ & T_{\min} \le T_{\text{in}}(k) \le T_{\max} \quad \forall k \end{array}$$

as

$$\begin{array}{ll} \min\limits_{T_{\text{in}}(k), \gamma \in \mathbb{R}} & 30\, p_{\text{off}} \sum\limits_{k \in I_{\text{off}}} q(k) + 30\, p_{\text{on}} \sum\limits_{k \in I_{\text{on}}} q(k) + p_d \gamma \\ \text{subject to} & \boxed{q(k) \le \gamma} \quad \forall k \in I_{\text{on}} \\ & q(k) = q_{\text{loss}}(T_{\text{in}}, T_{\text{out}}) + q_{\text{w}}(T_w) \quad \forall k \\ & T_w(k+1) = f(T_w(k), T_{\text{in}}) \quad \forall k \\ & T_{\min} \le T_{\text{in}}(k) \le T_{\max} \quad \forall k \end{array}$$

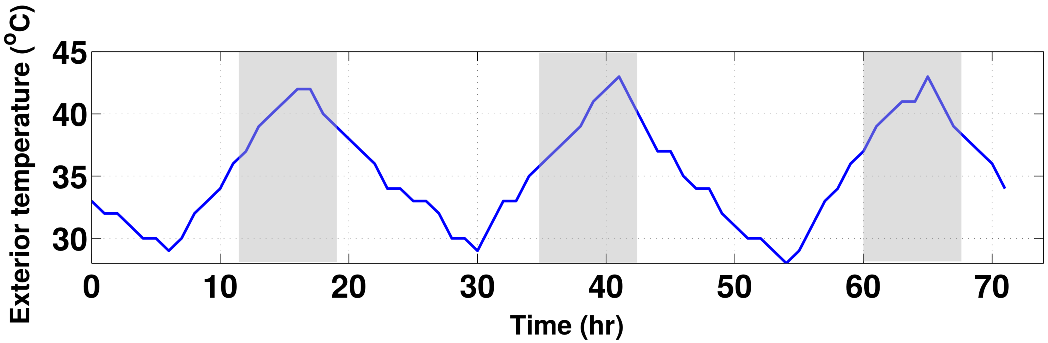
- For fixed γ, the reformulated problem can be solved by **Dynamic Programming**.
- γ is a scalar, so we use **bisection** over γ.

Building's Parameters and Outside Temperature in User's Problem

Building's parameters

wall's width	thermal diffusivity	thermal resistance	thermal capacity
0.4 (m)	8.3×10^{-7} (m^2/s)	0.0015 (K/W)	45 (Wm/K)

External temperature of three typical days in Phoenix, AZ

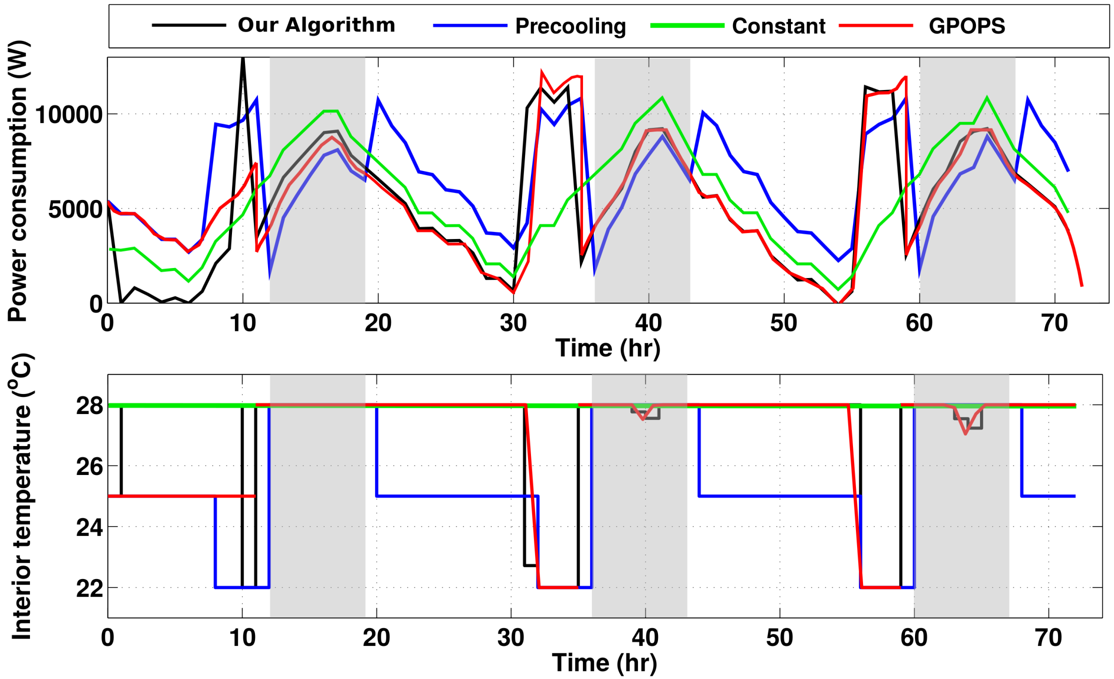


On-peak, off-peak & demand prices from Arizona power company APS

	On-peak $(\frac{\$}{kWh})$	Off-peak $(\frac{\$}{kWh})$	Demand $(\frac{\$}{kWh})$
APS	0.089	0.044	13.50

Our Algorithm Is A Good Way To Reduce Electricity Bills

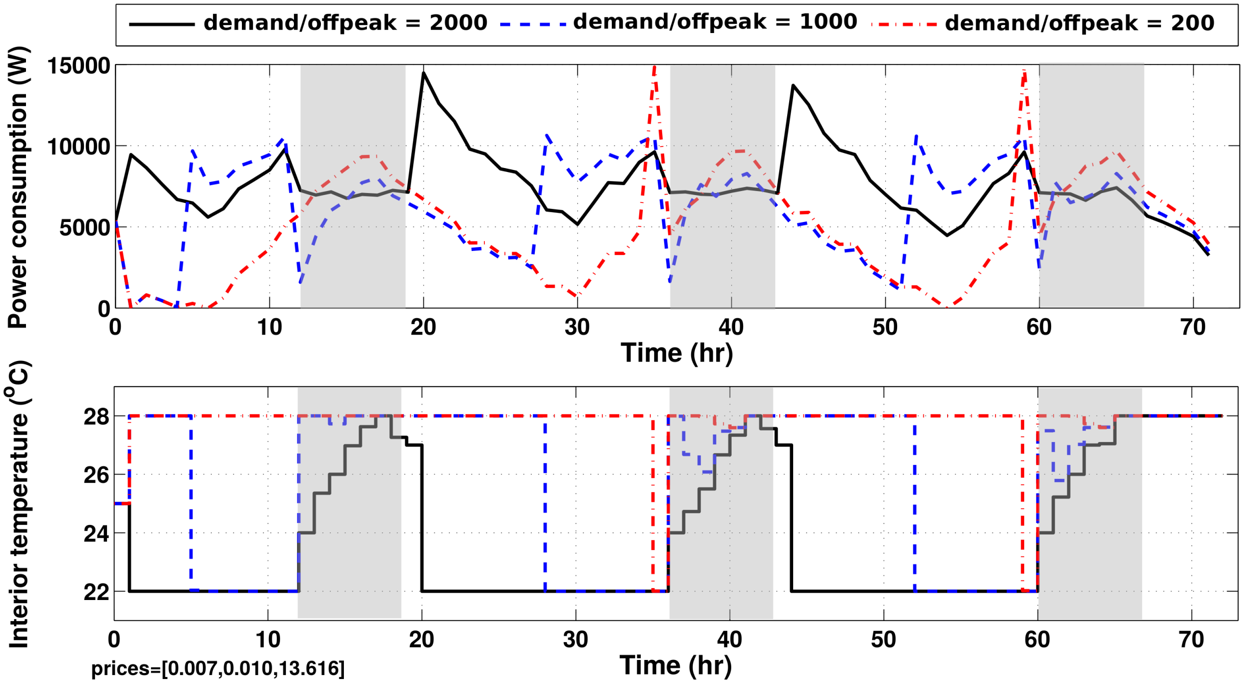
User's consumption and interior temperature using prices from Arizona Public Service



Temperature setting	Our algorithm	GPOPS	Pre-cooling	Constant
Monthly bill	**365.8**$	370.3$	392.3$	394.2$

Increasing $\frac{p_d}{p_{\text{off}}}$ Helps Reducing Max Consumption during on-peak

Weight of **demand** price relative to **on-peak** & **off-peak** prices affects maximum consumption



Peak is only suppressed during the on-peak hours

Pricing Optimization Problem Revisited!

To set prices $p_{\text{on}}, p_{\text{off}}, p_d$, power companies can solve:

- **Objective:** minimize the cost of generating electricity

$$\min_{p_{\text{on}}, p_{\text{off}}, p_d} \left(\underbrace{a \sum_k g(k) dt}_{\text{fuel cost}} + \underbrace{b \sup_{k \in \text{ on-peak period}} g(k)}_{\text{cost of building generators}} \right)$$

  - $g(k)$: power (kW) generated at time t

- **Constraint:**
  1. Equality of **generation** $(g(t))$ and **power** $(q_{\text{user}}(t))$ consumed by users:

$$\boxed{g(k) = q_{\text{user}}(k, p_{\text{off}}, p_{\text{on}}, p_d) \quad \forall t}$$

- **Variables:** on-peak, off-peak and demand prices: $p_{\text{on}}, p_{\text{off}}, p_d$

We Solved Power Company's Problem By A Descent Algorithm

We solved the power company's problem with a single user by

- Applying a **descent algorithm** to optimize over prices $p_{\text{on}}, p_{\text{off}}, p_d$
- Used **Dynamic Programming** at each iteration of the descent algorithm to find an optimal power generation

Initialize prices $p = [p_{\text{on}}, p_{\text{off}}, p_d]$
while ***Cost$_{new}$*** $-$ ***Cost$_{old}$*** $> \epsilon$ **do**
  Find a descent direction by evaluating the cost at a 7-point stencil centered at p
    For each price, solve user's problem using bisection & dynamic programming
  Update the best price $p_{\text{on}}, p_{\text{off}}, p_d$ and best cost
end

6.3% Reduction in Generation Cost For Salt River Project

- Comparison of generation costs for 3 days, using Salt River Project's prices and optimal prices:

Strategy	$[p_{\text{off}}$ (\$/kWh), p_{on} (\$/kWh), p_d (\$/kW)$]$	Generation cost
Our Algorithm	$[0.0820, 0.1080, 54.004]$	**83.33$**
SRP	$[0.0572, 0.0814, 59.760]$	89.00$

- Result is **6.3% reduction** in generation cost which corresponds to $\simeq$ **2 M$ saving per month.**

Integration of Renewables Has Minor Effect On Costs & Peaks

- We solved the power company's problem when **50% of users** have access to local **solar generation**

Users	Optimal prices $[p^\star_{\text{off}}, p^\star_{\text{on}}, p^\star_d]$	Electricity Bill	Max power used
Solar & Non-solar	$[0.089, 0.115, 51.988]$	$\$$ 50.05 $\$$ 84.71	6.1947 kW 8.6787 kW
Single Non-solar	$[0.081, 0.108, 54.004]$	$\$$ 83.33	8.3008 kW
Single Solar	$[0.088, 0.118, 58.556]$	$\$$ 54.31	6.1916 kW

- When optimal prices are used, 50% increase in renewables causes $<$ **2% change** in the bill of nonsolar users

- When SRP prices are used, 50% increase in renewables causes **8% change** in the bill of nonsolar users

Conclusions

- We defined a new model for **optimal behavior** of a user who minimizes his electricity bill based on given prices

  ↳ Optimal thermostat programming

- Used our model to define a framework for optimization of electricity prices for rational users

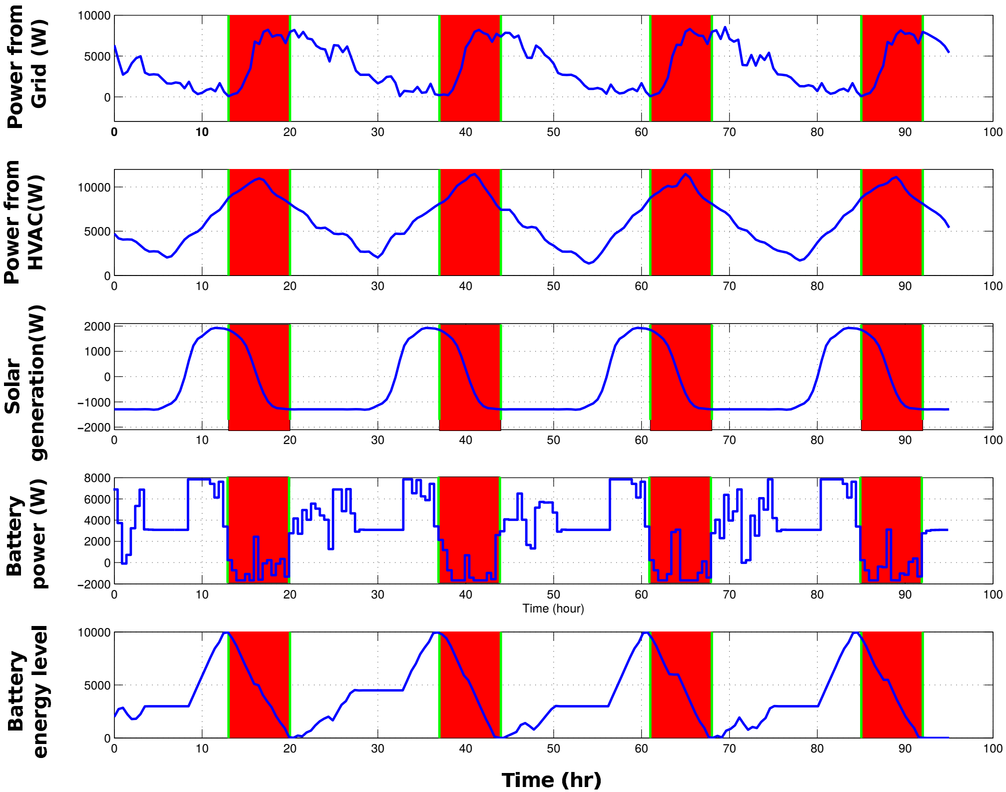
  ↳ Objective is to **minimize the cost of generation** while generation equals consumption

- We proposed prices which induce **30% reduction** in peak load and more than **6% reduction** in generation cost

- We would like to thank Salt River Project power company of Arizona for funding this research and providing data

Ongoing Work: Peak Load Reduction Using Batteries

- Incorporating **residential batteries**, such as Tesla's Powerwall in our user's models to reduce demand charges



Optimal residential battery control for minimizing electricity bill

Ongoing Work: Improving Our Model For Generation Cost

We used the following model for cost of generation:

$$\underbrace{a \int_{t=0}^{t=24} g(t)dt}_{\text{fuel cost}} + \underbrace{b \sup_{t\in \text{ on-peak period}} g(t)}_{\text{cost of building generators}}$$

- An improved model will include the costs associated with
  - ↳ Fuel cost of various types of generating units
  - ↳ Unit commitment: Cost for bringing each generating unit online
  - ↳ Arbitrage: Selling/buying from electricity spot market