Optimal Thermostat Programming and Electricity Prices for Customers with Demand Charges

Reza Kamyar and Matthew Peet
Cybernetic Systems and Controls Laboratory

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Power Companies Pay For Fuel & Generators

A simplified model for cost of generating electricity is a combination of:

1. **Cost of fuel** required to generate the total energy (kWh) consumed by users

   A common model is:
   \[
   \text{cost of fuel} = a \int q(t) \, dt
   \]
   
   \(q(t)\) (kW): power consumed by users, \(a\) ($/kWh): cost of fuel required to produce the next kWh

2. **Cost of building & maintaining generators** to accommodate for the maximum total power (kW) consumed by users

   A simple model can be:
   \[
   \text{Cost of building & maintaining generators} = b \sup_{t \in \text{on-peak}} q(t)
   \]
   
   \(b\) ($/kW): cost of installing the next kW of generating capacity
Current Pricing Strategies Do Not Charge For Max Power

- Most power companies use **flat** or **Time-of-Use (ToU)** pricing

  ➔ **Flat pricing**: Charges are independent of when energy is used

  \[
  \int q_1(t) dt \times \frac{\text{price}}{\text{kWh}} = \int q_2(t) dt \times \frac{\text{price}}{\text{kWh}}
  \]

  Electricity bills independent of \(q_{1\text{ max}}\) & \(q_{2\text{ max}}\)

  ➔ **ToU pricing**: Does not explicitly charge for max power used

  Elect. Bill = \(p_{\text{off}} \int_{\text{off-peak}} q(t) dt \)
  \[+ \ p_{\text{on}} \int_{\text{on-peak}} q(t) dt\]

  Large peak does not necessarily result in a large monthly bill
Current Pricing Strategies Are Problematic For Power Companies

- **Fact 1:** The ratio of maximum power used per year to average power used per year is setting records in the US!

  Partially due to increasing integration of renewables, e.g., solar

![Graph showing peak to average demand over years]

- **Fact 2:** Integration of renewables **does NOT** affect maximum power consumption, but reduces the total power sold by power companies ⇒ revenue decreases

- **Consequence:** Power companies won’t have enough revenue to supply for electricity without raising the prices

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Demand Charge: A Solution To The Revenue Problem

- **Demand charge**: A monthly charge proportional to the maximum power consumed by the user during the on-peak hours of a month.
- A combination of off-peak, on-peak and demand charges can differentiate between “good” and “bad” user behavior.

![Diagram showing power consumption and charging periods]

**Electricity Bill** =

\[
p_{\text{off}} \int_{t \in \text{off-peak}} q(t) \, dt + p_{\text{on}} \int_{t \in \text{on-peak}} q(t) \, dt + p_d \sup_{t \in \text{on-peak}} q(t)
\]

- on-peak period charge
- off-peak period charge
- demand charge

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How Can Power Companies Optimize Their Prices?

Power companies can solve the following optimization problem:

- **Objective:** minimize the cost of generating electricity

\[
\min_{p_{on}, p_{off}, p_d} \left( \underbrace{a \int_{t=0}^{t=24} g(t) \, dt}_{\text{fuel cost}} + \underbrace{b \sup_{t \in \text{on-peak period}} g(t)}_{\text{cost of building generators}} \right)
\]

- \( g(t) \): power (kW) generated at time \( t \)
- \( a \) (\$/kWh): cost of fuel required to produce the next kWh
- \( b \) (\$/kW): cost of installing the next kW of production capacity

- **Constraint:**
  - Equality of generation \( g(t) \) and power \( q_{user}(t) \) consumed by users:

\[
g(t) = q_{user}(t, p_{off}, p_{on}, p_d) \quad \forall t
\]

- **Variables:** on-peak, off-peak and demand prices: \( p_{on}, p_{off}, p_d \)
To optimize electricity prices, we need a model for users’ power consumption which;

1. Predicts how much electricity would a rational user consume, given the prices

**Question**: How can a rational user reduce his electricity bill?

- One way is to reduce HVAC load by using Energy storage
  1. Energy storage in residential batteries allows users to shift peaks from high-demand hours to another hours
  2. Using walls/floors as thermal energy storage: A free alternative to batteries
Precooling: As Time-of-Use Strategy To Reduce Bills

Precooling exploits thermal energy storage in walls to shift loads:

- Cool down walls/floors when electricity is cheap

- Cold walls will reduce the load on HVAC during on-peak hours - thus reducing the electricity bill
Precooling Fails When Demand Charges Are Applied

- Precooling does **NOT** reduce max power consumption. **Why?**
  1. Thermal storage in the walls **depletes** before the end of the on-peak period
  2. Then HVAC will remain as the **only** cooling mechanism
  3. At the end of on-peak period, same load will be on HVAC as if no precooling had occurred

- When demand charges exist, thermostat programming is difficult!
  - Thermal storage is governed by the heat equation - A PDE
  - Heat equation inherently has **latency**, thus a good strategy may involve counter-intuitive temperature settings

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How Do Thermostat Settings Affect Energy Consumption?

Power consumed by user is a combination of heat loss to outside and heat given to/taken from interior walls

\[ q_{\text{user}}(t) = q_{\text{loss}}(t) + q_{\text{wall}}(t) \quad \forall k \]

- Heat loss \( q_{\text{loss}}(t) \) is modeled by a linear heat sink and can be controlled by interior temperature \( T_{\text{in}} \):
  \[ q_{\text{loss}}(t) = \frac{T_{\text{out}}(t) - T_{\text{in}}(t)}{R_w} \]

  \( T_{\text{out}} \): Outside temperature \quad \( R_w \): thermal resistance

- Heat thru walls \( q_{\text{wall}}(k) \) is modeled by the Heat equation (PDE):
  \[ \frac{\partial T_w(t, x)}{\partial t} = \alpha \frac{\partial^2 T_w(t, x)}{\partial x^2} \]

  \[ q_{\text{wall}}(k) = 2C_w \frac{\partial T_w}{\partial x}(t, 0) \]
How Do Rational Users Minimize Their Bill?

User can solve a discrete-time thermostat programming problem with

- **Objective:** minimize the electricity bill
  \[
  \min_{T_{\text{in}}(k)} \left( 30 p_{\text{off}} \sum_{k \in I_{\text{off}}} q_{\text{user}}(k) + 30 p_{\text{on}} \sum_{k \in I_{\text{on}}} q_{\text{user}}(k) + p_d \sup_{k \in I_{\text{on}}} q_{\text{user}}(k) \right)
  \]
  - OFF-peak period charge
  - ON-peak period charge
  - demand charge

- **Constraints:**
  1. Interior temperature with a certain bound:
     \[ T_{\text{min}} \leq T_{\text{in}}(k) \leq T_{\text{max}} \quad \forall k \]
  2. Energy conservation:
     \[ q_{\text{user}}(k) = q_{\text{loss}}(T_{\text{in}}(k), T_e(k)) + q_{\text{wall}}(T_w(x, k)) \quad \forall k \]
  3. Discretized heat dynamics:
     \[ T_w(k + 1) = A T_w(k) + B T_{\text{in}}(k) \]

- **Variables:** Interior temperature \( T_{\text{in}}(k) \) over time
A Reformulation of User’s Problem Can Be Solved By Dynamic Programming

- We reformulate the user’s problem

\[
\begin{align*}
\min_{T_{in}(k)} & \quad 30 \sum_{k \in I_{off}} q(k) + 30 \sum_{k \in I_{on}} q(k) + \max_{k \in I_{on}} p_d \sup_{k \in I_{on}} q(k) \\
\text{subject to} & \quad q(k) = q_{loss}(T_{in}, T_{out}) + q_w(T_w) \quad \forall k \\
& \quad T_w(k + 1) = f(T_w(k), T_{in}) \quad \forall k \\
& \quad T_{min} \leq T_{in}(k) \leq T_{max} \quad \forall k
\end{align*}
\]

as

\[
\begin{align*}
\min_{T_{in}(k), \gamma \in \mathbb{R}} & \quad 30 \sum_{k \in I_{off}} q(k) + 30 \sum_{k \in I_{on}} q(k) + p_d \gamma \\
\text{subject to} & \quad q(k) \leq \gamma \quad \forall k \in I_{on} \\
& \quad q(k) = q_{loss}(T_{in}, T_{out}) + q_w(T_w) \quad \forall k \\
& \quad T_w(k + 1) = f(T_w(k), T_{in}) \quad \forall k \\
& \quad T_{min} \leq T_{in}(k) \leq T_{max} \quad \forall k
\end{align*}
\]

- For fixed \( \gamma \), the reformulated problem can be solved by Dynamic Programming.

- \( \gamma \) is a scalar, so we use bisection over \( \gamma \).
### Building’s Parameters and Outside Temperature in User’s Problem

#### Building’s parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>wall’s width</td>
<td>0.4 (m)</td>
</tr>
<tr>
<td>thermal diffusivity</td>
<td>$8.3 \times 10^{-7}$ (m$^2$/s)</td>
</tr>
<tr>
<td>thermal resistance</td>
<td>0.0015 (K/W)</td>
</tr>
<tr>
<td>thermal capacity</td>
<td>45 (Wm/K)</td>
</tr>
</tbody>
</table>

#### External temperature of three typical days in Phoenix, AZ

![Graph showing temperature vs. time](image)

#### On-peak, off-peak & demand prices from Arizona power company

<table>
<thead>
<tr>
<th></th>
<th>On-peak ($\dfrac{$}{kWh}$)</th>
<th>Off-peak ($\dfrac{$}{kWh}$)</th>
<th>Demand ($\dfrac{$}{kWh}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>APS</td>
<td>0.089</td>
<td>0.044</td>
<td>13.50</td>
</tr>
</tbody>
</table>

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Our Algorithm Is A Good Way To Reduce Electricity Bills

User’s consumption and interior temperature using prices from Arizona Public Service

![Graph showing power consumption and interior temperature over time.](image)

<table>
<thead>
<tr>
<th>Temperature setting</th>
<th>Our algorithm</th>
<th>GPOPS</th>
<th>Pre-cooling</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly bill</td>
<td><strong>365.8$</strong></td>
<td>370.3$</td>
<td>392.3$</td>
<td>394.2$</td>
</tr>
</tbody>
</table>
Increasing $\frac{p_{d}}{p_{off}}$ Helps Reducing Max Consumption during on-peak

Weight of **demand** price relative to **on-peak & off-peak** prices affects maximum consumption

Peak is only suppressed during the on-peak hours
Pricing Optimization Problem Revisited!

To set prices $p_{on}$, $p_{off}$, $p_d$, power companies can solve:

- **Objective:** minimize the cost of generating electricity

\[
\min_{p_{on}, p_{off}, p_d} \left( a \sum_k g(k) dt + b \sup_{k \in \text{on-peak period}} g(k) \right)
\]

- **Constraint:**
  1. Equality of **generation** ($g(t)$) and **power** ($q_{user}(t)$) consumed by users:

\[
g(k) = q_{user}(k, p_{off}, p_{on}, p_d) \quad \forall t
\]

- **Variables:** on-peak, off-peak and demand prices: $p_{on}$, $p_{off}$, $p_d$
We solved the power company’s problem with a single user by

- Applying a **descent algorithm** to optimize over prices $p_{on}, p_{off}, p_d$
- Used **Dynamic Programming** at each iteration of the descent algorithm to find an optimal power generation

**Initialize prices**

$\begin{align*}
\text{while } \text{Cost}_{\text{new}} - \text{Cost}_{\text{old}} > \epsilon \text{ do} \\
\quad \text{Find a descent direction by evaluating the cost at a 7-point stencil centered at } p \\
\quad \text{For each price, solve user’s problem using bisection & dynamic programming} \\
\quad \text{Update the best price } p_{on}, p_{off}, p_d \text{ and best cost} \\
\text{end}
\end{align*}$
6.3% Reduction in Generation Cost For Salt River Project

- Comparison of generation costs for 3 days, using Salt River Project’s prices and optimal prices:

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$p_{off}$ ($/kWh), $p_{on}$ ($/kWh), $p_d$ ($/kW)$</th>
<th>Generation cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our Algorithm</td>
<td>[0.0820, 0.1080, 54.004]</td>
<td>83.33$</td>
</tr>
<tr>
<td>SRP</td>
<td>[0.0572, 0.0814, 59.760]</td>
<td>89.00$</td>
</tr>
</tbody>
</table>

- Result is 6.3% reduction in generation cost which corresponds to $\approx 2 \text{ M$ saving per month.}$
Integration of Renewables Has Minor Effect On Costs & Peaks

- We solved the power company’s problem when **50% of users** have access to local **solar generation**

<table>
<thead>
<tr>
<th>Users</th>
<th>Optimal prices</th>
<th>Electricity Bill</th>
<th>Max power used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar &amp; Non-solar</td>
<td>$[0.089, 0.115, 51.988]$</td>
<td>$$50.05$</td>
<td>$6.1947 kW$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$$84.71$</td>
<td>$8.6787 kW$</td>
</tr>
<tr>
<td>Single Non-solar</td>
<td>$[0.081, 0.108, 54.004]$</td>
<td>$$83.33$</td>
<td>$8.3008 kW$</td>
</tr>
<tr>
<td>Single Solar</td>
<td>$[0.088, 0.118, 58.556]$</td>
<td>$$54.31$</td>
<td>$6.1916 kW$</td>
</tr>
</tbody>
</table>

- When optimal prices are used, 50% increase in renewables causes **< 2% change** in the bill of nonsolar users

- When SRP prices are used, 50% increase in renewables causes **8% change** in the bill of nonsolar users
Conclusions

- We defined a new model for **optimal behavior** of a user who minimizes his electricity bill based on given prices
  - Optimal thermostat programming

- Used our model to define a framework for optimization of electricity prices for rational users
  - Objective is to **minimize the cost of generation** while generation equals consumption

- We proposed prices which induce **30% reduction** in peak load and more than **6% reduction** in generation cost

- We would like to thank Salt River Project power company of Arizona for funding this research and providing data
Ongoing Work: Peak Load Reduction Using Batteries

- Incorporating **residential batteries**, such as Tesla’s Powerwall in our user’s models to reduce demand charges.

**Optimal residential battery control for minimizing electricity bill**

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Ongoing Work: Improving Our Model For Generation Cost

We used the following model for cost of generation:

\[ a \int_{t=0}^{t=24} g(t) \, dt + b \sup_{t \in \text{on-peak period}} g(t) \]

- An improved model will include the costs associated with:
  - **Fuel cost** of various types of generating units
  - **Unit commitment**: Cost for bringing each generating unit online
  - **Arbitrage**: Selling/buying from electricity spot market