

# Multi-objective Dynamic Programming for Constrained Optimization of Non-separable Objective Functions with Application in Energy Storage

Reza Kamyar and Matthew M. Peet

**Abstract**— We propose a multi-objective optimization algorithm for optimal energy storage by residential customers using Li-Ion batteries. Our goal is to quantify the benefits of optimal energy storage to solar customers whose electricity bills consist of both Time of Use charges (\$/kWh, with different rates for on-peak and off-peak hours) and demand charges (\$/kW, proportional to the peak rate of consumption in a month). We first define our energy storage optimization problem as minimization of the monthly electricity bill subject to certain constraints on the energy level and the charging/discharging rate of the battery, while accounting for battery’s degradation due to cycling and depth of discharge. We solve this problem by constructing a sequence of parameterized multi-objective dynamic programs whose sets of non-dominated solutions are guaranteed to contain an optimal solution to our energy storage problem. Unlike the standard formulation of our energy storage problem, each of the parameterized optimization problems satisfy the principle of optimality - hence can be solved using standard dynamic programming algorithms. Our numerical case studies on a wide range of load profiles and various pricing plans show that optimal energy storage using Tesla’s Powerwall battery can reduce the monthly electricity bill by up to 52% relative to the case where no energy storage is used.

## I. INTRODUCTION

One concern of electric utilities is that rapid integration of renewables such as distributed solar generation may change customers’ consumption behavior in ways that current generating units cannot accommodate for these changes. The demand peaks are of particular concern to regulated public utilities as these utilities are required to maintain reserve generating capacity as determined by the demand peak. According to a recent report [1] by Arizona Public Services (APS), the demand peak in Arizona is projected to increase by 40% over a period of next 15 years (See Fig. 1). In order to cover the cost of increasing the generating capacity and influence the consumption behavior of rational customers, Arizona’s utilities Salt River Project (SRP) and APS have recently incorporated *demand charges* in some of their pricing plans. Demand charges are proportional to the maximum rate at which the electricity is consumed by the customer during the on-peak hours within a month. Thus, in this new form of pricing, the amount paid by the customer within a month is

$$J(q) := 30 p_{\text{on}} \int_{T_{\text{on}}} q(t) dt + 30 p_{\text{off}} \int_{T_{\text{off}}} q(t) dt + p_d \sup_{t \in T_{\text{on}}} q(t), \quad (1)$$

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where  $q(t)$  is the power supplied by the utility company. The first two terms in (1) are Time of Use (ToU) charges, where  $p_{\text{on}}$  (\$/kWh) and  $p_{\text{off}}$  (\$/kWh) are the on-peak and off-peak prices. The third term is the monthly demand charge, where  $p_d$  (\$/kW) is called the demand price. In (1),  $T_{\text{on}}$  and  $T_{\text{off}}$  denote the on-peak and off-peak periods respectively.

One approach for rational customers to reduce their consumption peaks (as a response to the integration of demand charges) is energy storage [2]. Several papers have studied optimal use of battery storage for residential customers [3], [4], [5], [6]. Majority of the existing papers use Model Predictive Control [7], dynamic programming-based algorithms [4], [5] or neural networks [6] to find optimal storage programs under real-time or ToU pricing. In [8], the problem of optimal scheduling of battery and other controlled appliances is coupled with a social welfare optimization problem at the utility level to find optimal battery schedules and optimal real-time prices. Although optimal battery scheduling is a well-developed research area, to the best of our knowledge, there has been no studies on optimal scheduling of residential-sized batteries in the presence of demand charges.

From the mathematical standpoint, the presence of demand charges poses a significant challenge in solving the underlying optimization problem. In particular, because of the existence of the term  $\sup q(t)$  (a *non-separable* term with respect to time) in the electricity bill defined in (1), the problem  $\min J(q)$  violates the principle of optimality [9]. The principle of optimality provides sufficient conditions for optimal control algorithms such as dynamic programming and Hamiltonian-based algorithms to converge to an optimal solution. While works such as [10], [11] have included some forms of demand charges in their storage optimization problems, they have not addressed the problem of inseparability

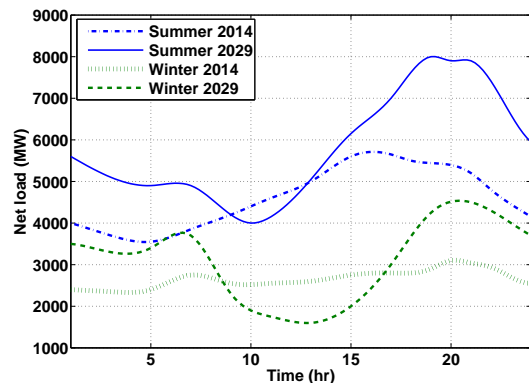


Fig. 1. Net retail load for typical summer and winter days in Arizona for years 2014 and 2029 (projected)

of the objective function and violation of the principle of optimality.

In this paper, we establish a provably convergent algorithm for optimal battery scheduling for residential customers, under a combined ToU and demand pricing plan. We achieve this in two steps: First, we replace  $\sup q(t)$  in the objective function with an  $L_p$  approximation of  $q(t)$  for some large  $p$ . Then, we construct a sequence of multi-objective optimization problems (similar to [12], [13]) whose sets of non-dominated solutions contain an optimal solution to the residential battery optimization as  $p$  tends to infinity. The objective functions of these multi-objective problems are separable with respect to time - hence can be minimized using the existing dynamic programming algorithms. In a number of numerical case studies, we applied our algorithm to the residential battery optimization problem for a wide range of customer's load, while considering the effects of solar generation and battery's degradation over 5 years. We used Tesla's Powerwall battery as the source of energy storage. Our numerical analysis shows that optimal energy storage can reduce the monthly electricity bill by up to 52% using SRP's pricing plan [14]. Moreover, we observed that under a combined ToU and demand pricing, optimal energy storage is most effective (in terms of saving over 5 years) for those customers who have the highest monthly peak load within our range of analysis.

## II. OPTIMAL ENERGY STORAGE PROBLEM

In this section, we first define a model describing the amount of energy stored in the battery and the battery degradation due to cycling and depth of discharge. We then use this model to define our residential battery storage optimization problem under a mixed ToU and demand pricing plan.

### A. A Model for Battery Storage and Degradation

To model the amount of energy stored in the battery, we use the difference equation

$$e(d, k+1) = e(d, k) + \eta u(d, k)\Delta t, \quad (2)$$

where  $e(d, k)$  denotes the amount of energy stored in the battery at time  $t = k\Delta t$  of day  $d$ ,  $u(d, k)$  denotes the charging/discharging (+/-) rate at time  $t = k\Delta t$  of day  $d$ , and  $\eta$  denotes the charging efficiency. We denote the initial capacity of the battery by  $C_I$ , meaning that the initial energy level  $e(0, 0)$  is bounded by  $C_I$ . Moreover, we denote the maximum charging and discharging rates by  $\bar{u}$  and  $\underline{u}$ , meaning that  $u(k) \in [\underline{u}, \bar{u}]$  for all  $k$ .

We model the capacity of the battery at day  $d$  and time-step  $k$  as

$$c_b(d, k) = (1 - \beta(d, k))C_I, \quad (3)$$

where  $\beta(d, k)$  denotes the battery's degradation rate. We model the degradation rate as the sum of cycling degradation and depth of discharge degradation:

$$\beta(d, k) = \underbrace{\alpha n(d, k)}_{\text{cycling degradation}} + \underbrace{\sum_{k=0}^T \gamma(d, k)}_{\text{depth of discharge degradation}}. \quad (4)$$

In (4),  $n(d, k)$  is the total number of cycles completed till time-step  $k$  of day  $d$ . We calculate the number of cycles recursively as

$$n(d, k+1) = n(d, k) + \frac{\eta \Delta t |u(d, k)|}{2c_b(d, k)}. \quad (5)$$

In (4),  $\gamma(d, k)$  is the depth of discharge degradation at time-step  $k$  of day  $d$ . We use a model similar to the Li-Ion battery model in [15] to calculate  $\gamma(d, k)$  as

$$\gamma(d, k) = \begin{cases} X \zeta(d, k) + Y & \zeta(d, k) \geq 0.7 \\ 0 & \text{otherwise} \end{cases}, \quad (6)$$

where  $\zeta(d, k)$  is the depth of discharge at time-step  $k$  and is calculated as

$$\zeta(d, k) = \frac{c_b(d, k) - e(d, k)}{c_b(d, k)}.$$

In this paper, we consider Tesla's Powerwall as the battery used by the residential customer. As declared by Tesla, this battery degrades to 80% of its initial capacity  $C_I = 10$  kWh after completing 5000 round-trip cycles. To capture the 20% drop in the capacity, we set the parameters  $\alpha$ ,  $X$  and  $Y$  in (4) and (6) to  $1 \times 10^{-4}$ ,  $5.8 \times 10^{-6}$  and  $-4 \times 10^{-6}$  respectively.

### B. Problem Statement: Residential Battery Storage Optimization

We define our battery storage optimization problem as minimizing the monthly electricity bill subject to constraints on the battery's energy-level and charging/discharging rates. To define the electricity charges within a month, we divide each day into two off-peak periods from 12 AM to  $t_{\text{on}}$  and  $t_{\text{off}}$  to 12 AM. Let us denote the electricity price during the off-peak periods by  $p_{\text{off}}$  (\$/kWh). The on-peak period starts at  $t_{\text{on}}$  and ends at  $t_{\text{off}}$ . We denote the electricity price during this period by  $p_{\text{on}}$  (\$/kWh). In addition to the on-peak and off-peak charges, we assume that customers are charged proportional to their maximum rate of consumption during the on-peak hours in a month. We denote the proportionality ratio by  $p_d$  (\$/kW) and call it the *demand price*. Given the prices  $p_{\text{off}}$ ,  $p_{\text{on}}$  and  $p_d$ , the ToU charge in a month is calculated as

$$J_t(q, p_{\text{off}}, p_{\text{on}}) = \sum_{d=1}^{N=30} \left( p_{\text{off}} \sum_{k=0}^{t_{\text{on}}-1} q(d, k)\Delta t + p_{\text{on}} \sum_{k=t_{\text{on}}}^{t_{\text{off}}-1} q(d, k)\Delta t + p_{\text{off}} \sum_{k=t_{\text{off}}}^{24/\Delta t} q(d, k)\Delta t \right),$$

and the demand charge is calculated as

$$J_d(q, p_d) = p_d \sup_{\substack{k \in \{t_{\text{on}}, \dots, t_{\text{off}}-1\} \\ d \in \{1, \dots, 30\}}} q(d, k), \quad (7)$$

where  $q(d, k)$  is the power supplied by the grid at time-step  $k$  of day  $d$ .

We now define the problem of optimal residential battery

storage for a month as

$$\begin{aligned}
& \min_{\substack{u(d,k), \beta(d,k) \in \mathbb{R} \\ e(d,k), n(d,k) \in \mathbb{R}}} J_t(q, P_{\text{off}}, P_{\text{on}}) + J_d(q, p_d) \quad \text{subject to} \\
& q(d, k) = q_a(d, k) - q_s(d, k) + u(d, k) \quad \text{for } d \in \Phi_d \text{ and } k \in \Phi_k \\
& e(d, k+1) = e(d, k) + \eta u(d, k) \Delta t \quad \text{for } d \in \Phi_d \text{ and } k \in \Phi_k \\
& e(d, 0) = e\left(d-1, \frac{24}{\Delta t}\right) + \eta u\left(d-1, \frac{24}{\Delta t}\right) \Delta t \quad \text{for } d \in \Phi_d \\
& 0 \leq e(d, k) \leq (1 - \beta(d, k)) C_I \quad \text{for } d \in \Phi_d \text{ and } k \in \Phi_k \\
& \beta(d, k) = \alpha n(d, k) + \sum_{k=0}^T \gamma(d, k) \quad \text{for } d \in \Phi_d \text{ and } k \in \Phi_k \\
& n(d, k+1) = n(d, k) + \frac{\eta \Delta t |u(d, k)|}{2c_b(d, k)} \quad \text{for } d \in \Phi_d \text{ and } k \in \Phi_k \\
& \underline{u} \leq u(d, k) \leq \bar{u} \quad \text{for } d \in \Phi_d \text{ and } k \in \Phi_k \\
& e(0, 0) = e_0, n(0, 0) = 0, \quad (8)
\end{aligned}$$

where we define  $\Phi_d := \{1, \dots, N\}$  and  $\Phi_k := \{0, \dots, 24/\Delta t\}$ . The second line of Problem (8) constrains the sum of the power,  $u$ , given to the battery and the power,  $q_a$ , consumed by the appliances to be equal to the total power supplied by the grid and the solar Photovoltaics. Lines 3 and 4 enforce the change in the battery's energy level to be proportional to the power given to or take from the battery. Line 5 constrains the energy level of the battery to stay below the battery's capacity. Lines 6 and 7 model the battery's degradation. Finally, Line 8 constrains the charging/discharging rate to stay within a pre-specified bound determined by the manufacturer.

### III. SOLUTION METHODOLOGY: MULTI-OBJECTIVE DYNAMIC PROGRAMMING

The residential battery optimization defined in (8) is an instance of the general optimal control problem

$$\begin{aligned}
J^*(z) & := \min_{u_k \in \mathbb{R}^m, x_k \in \mathbb{R}^n} J(x_0, u_0, \dots, x_{N-1}, u_{N-1}, x_N) \\
\text{subject to } & x_{k+1} = f(x_k, u_k), x_0 = z \quad \text{for } k = 0, \dots, N \\
& x_k \in X \quad \text{for } k = 1, \dots, N \\
& u_k \in U \quad \text{for } k = 0, \dots, N-1, \quad (9)
\end{aligned}$$

where  $z \in \mathbb{R}^n$  is a given initial state, and  $f: \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ ,  $X \subset \mathbb{R}^n$  and  $U \subset \mathbb{R}^m$  are known. It can be shown that if the objective function,  $J$ , of Problem (9) is *time-separable*, i.e., there exist maps  $\psi_k: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}$  for  $k = 0, \dots, N-1$  and  $\psi_N: \mathbb{R}^n \rightarrow \mathbb{R}$  such that

$$\begin{aligned}
J(x_0, u_0, \dots, x_{N-1}, u_{N-1}, x_N) & = \psi_0\left(x_0, u_0, \psi_1\left(x_1, u_1, \right. \right. \\
& \left. \left. \psi_2\left(\dots, \psi_{N-1}\left(x_{N-1}, u_{N-1}, \psi_N(x_N)\right)\right)\right)\right), \quad (10)
\end{aligned}$$

for all  $u_k \in \mathbb{R}^m$ , where  $x_k = f(x_{k-1}, u_{k-1})$  for  $k = 1, \dots, N$ , then one can apply dynamic programming to Problem (9) to find  $J^*(z)$  for any  $z \in X$ . Dynamic programming uses the principle of optimality [9] to reduce Problem (9) to a finite sequence of optimization problems indexed by the time-steps  $k = 0, \dots, N$  (also known as Bellman's recursive formula):

$$\begin{aligned}
V_k(z) & = \min_{v \in U} (g(z, v) + V_{k+1}(f(z, v))) \quad \text{for } z \in X \text{ and } k \in \Phi_k \setminus N \\
V_N(z) & = h(z) \quad \text{for all } z \in X. \quad (11)
\end{aligned}$$

It can be shown that  $J(z) = V_0(z)$ . Moreover, at each time-step  $k$  and state  $z$ , the minimizer  $v$  in (11) is an optimal control. Unfortunately, the objective function of Problem (8) is not time-separable. Specifically, the supremum function in  $J_d$  as defined in (7), can not be represented in the Form (10). Moreover, it can be shown that the objective function of Problem (8) violates the principle of optimality. As a result, dynamic programming will not necessarily converge to an optimal solution. In this section, we resolve this issue in two steps. First, we approximate the supremum function in  $J_d$  using the  $l_p$ -norm of  $q(d, k)$ , i.e.,

$$\sup_{\substack{k \in \{t_{\text{on}}, \dots, t_{\text{off}}-1\} \\ d \in \{1, \dots, 30\}}} q(d, k) \approx \sqrt[p]{\sum_{d=1}^{30} \sum_{k=t_{\text{on}}}^{t_{\text{off}}-1} q(d, k)^p}$$

for some large  $p \in \mathbb{N}$ . We then use Theorem 1 to define a multi-objective optimization problem whose set of *non-dominated* solutions contains an optimal solution to Problem (9) when  $p \rightarrow \infty$ . Before presenting the theorem, we define the set of non-dominated solutions as follows.

*Definition 1:* Consider the multi-objective problem

$$\begin{aligned}
& \min_{u_k \in \mathbb{R}^m, x_k \in \mathbb{R}^n} [D_1(x_0, u_0, \dots, u_{N-1}, x_{N-1}, x_N), \dots, \\
& \qquad \qquad \qquad D_M(x_0, u_0, \dots, u_{N-1}, x_{N-1}, x_N)] \\
& \text{subject to } x_{k+1} = f(x_k, u_k), x_0 = z \quad \text{for } k = 0, \dots, N \\
& \qquad \qquad \qquad x_k \in X \quad \text{for } k = 1, \dots, N \\
& \qquad \qquad \qquad u_k \in U \quad \text{for } k = 0, \dots, N-1,
\end{aligned}$$

for some  $z \in \mathbb{R}^n, X \subset \mathbb{R}^n, U \subset \mathbb{R}^m$ , where the maps

$$D_i: \mathbb{R}^n \times \mathbb{R}^m \times \dots \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^n \rightarrow \mathbb{R}$$

are known. A solution  $(x_0, u_0^*, \dots, u_{N-1}^*, x_{N-1}^*, x_N^*)$  to the multi-objective problem is called *non-dominated* if there exists no other feasible  $(x_0, u_0, \dots, u_{N-1}, x_{N-1}, x_N)$  such that  $D_i(x_0, u_0, \dots, u_{N-1}, x_{N-1}, x_N) \leq D_i(x_0, u_0^*, \dots, u_{N-1}^*, x_{N-1}^*, x_N^*)$  for  $i = 1, \dots, M$ , with strict inequality for at least one  $i$ .

We now present the main theorem.

*Theorem 1:* Suppose there exist maps  $f: \mathbb{R}^n \times \mathbb{R}^m, G: \underbrace{\mathbb{R} \times \dots \times \mathbb{R}}_{M\text{-times}} \rightarrow \mathbb{R}$  and  $D_i: \underbrace{(\mathbb{R}^n \times \mathbb{R}^m) \times \dots \times (\mathbb{R}^n \times \mathbb{R}^m)}_{(N-1)\text{-times}} \times \mathbb{R}^n \rightarrow \mathbb{R}$  for  $i = 1, \dots, M$  such that:

- I. (**Time-separability of  $D_i$** ) For each  $i \in \{1, \dots, M\}$ , there exist maps  $\psi_{i,j}: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}$  for  $j = 1, \dots, N-1$  and  $\psi_{i,N}: \mathbb{R}^n \rightarrow \mathbb{R}$  such that

$$\begin{aligned}
D_i(x_0, u_0, \dots, x_{N-1}, u_{N-1}, x_N) & = \psi_{i,0}\left(x_0, u_0, \psi_{i,1}\left(x_1, u_1, \right. \right. \\
& \left. \left. \psi_{i,2}\left(\dots, \psi_{i,N-1}\left(x_{N-1}, u_{N-1}, \psi_{i,N}(x_N)\right)\right)\right)\right) \quad (12)
\end{aligned}$$

for all  $u_k \in \mathbb{R}^m$ , where  $x_k = f(x_{k-1}, u_{k-1})$ ,  $k = 1, \dots, N$ ;

II. (**Backward monotonicity**  $D_i$ ) For each  $i \in \{1, \dots, M\}$  and any  $j \in \{1, \dots, N\}$ , if  $\psi_{i,j}$  in (12) satisfies

$$\begin{aligned} & \psi_{i,j} \left( x_j, u_j, \psi_{i,j+1} \left( x_{j+1}, u_{j+1}, \psi_{i,j+2} \left( \dots, \right. \right. \right. \\ & \quad \left. \left. \left. \psi_{i,N-1} (x_{N-1}, u_{N-1}, \psi_{i,N}(x_N)) \right) \right) \right) \\ & > \psi_{i,j} \left( x_j, v_j, \psi_{i,j+1} \left( y_{j+1}, v_{j+1}, \psi_{i,j+2} \left( \dots, \right. \right. \right. \\ & \quad \left. \left. \left. \psi_{i,N-1} (y_{N-1}, v_{N-1}, \psi_{i,N}(y_N)) \right) \right) \right) \end{aligned}$$

for all  $x_j \in \mathbb{R}^n$  and for some  $(u_j, \dots, u_{N-1})$  and  $(v_j, \dots, v_{N-1})$ , where  $x_k = f(x_{k-1}, u_{k-1})$  for  $k = j+1, \dots, N$  and  $y_k = f(y_{k-1}, v_{k-1})$  for  $k = j+1, \dots, N$  with  $y_j = x_j$ , then

$$\begin{aligned} & \psi_{i,j-1} \left( x_{j-1}, u_{j-1}, \psi_{i,j} \left( x_j, u_j, \psi_{i,j+1} \left( x_{j+1}, u_{j+1}, \psi_{i,j+2} \right. \right. \right. \\ & \quad \left. \left. \left. \left( \dots, \psi_{i,N-1} (x_{N-1}, u_{N-1}, \psi_{i,N}(x_N)) \right) \right) \right) \right) \\ & > \psi_{i,j-1} \left( x_{j-1}, u_{j-1}, \left( x_j, v_j, \psi_{i,j+1} \left( y_{j+1}, v_{j+1}, \psi_{i,j+2} \right. \right. \right. \\ & \quad \left. \left. \left. \left( \dots, \psi_{i,N-1} (y_{N-1}, v_{N-1}, \psi_{i,N}(y_N)) \right) \right) \right) \right) \end{aligned}$$

for all  $x_{j-1} \in \mathbb{R}^n$ ,  $u_{j-1} \in \mathbb{R}^m$ ;

III. (**Monotonicity of  $J$** )  $G, D_i$  and the objective function  $J$  in Problem (9) satisfy

$$J(x_0, u_0, \dots, x_{N-1}, u_{N-1}, x_N) =$$

$$G \left( D_1(x_0, u_0, \dots, x_{N-1}, u_{N-1}, x_N), \dots, D_M(x_0, u_0, \dots, x_{N-1}, u_{N-1}, x_N) \right), \quad (13)$$

where  $\frac{\partial G(D_1, \dots, D_M)}{\partial D_i} > 0$  for  $i = 1, \dots, M$ .

Then, an optimal solution to Problem (9) is a non-dominated solution of the multi-objective problem

$$\begin{aligned} & \min_{u_k \in \mathbb{R}^m, x_k \in \mathbb{R}^n} [D_1(x_0, u_0, \dots, u_{N-1}, x_{N-1}, x_N), \dots, \\ & \quad D_M(x_0, u_0, \dots, u_{N-1}, x_{N-1}, x_N)] \\ & \text{subject to } x_{k+1} = f(x_k, u_k), x_0 = z \quad \text{for } k = 0, \dots, N \\ & \quad x_k \in X \quad \text{for } k = 1, \dots, N \\ & \quad u_k \in U \quad \text{for } k = 0, \dots, N-1. \end{aligned}$$

See [13] for a proof. To apply Theorem 1 to our residential battery optimization problem, defined in (8), let us start by defining the maps  $G$  and  $D_i$  in theorem 1 as

$$\begin{aligned} D_1(q(1,0), \dots, q(30, 24/\Delta t)) & := \\ & \sum_{d=1}^{30} \left( p_{\text{off}} \sum_{k=0}^{t_{\text{on}}-1} q(d,k) \Delta t + p_{\text{on}} \sum_{k=t_{\text{on}}}^{t_{\text{off}}-1} q(d,k) \Delta t + p_{\text{off}} \sum_{k=t_{\text{off}}}^{24/\Delta t} q(d,k) \Delta t \right), \\ D_2(q(1, t_{\text{on}}), \dots, q(30, t_{\text{off}}-1)) & := p_d^p \sum_{d=1}^{30} \sum_{k=t_{\text{on}}}^{t_{\text{off}}-1} q(d,k)^p \quad (14) \end{aligned}$$

and

$$G(D_1, D_2) := D_1 + \sqrt[p]{D_2}, \quad (16)$$

where recall that  $\sqrt[p]{D_2}$  is the  $l_p$ -norm approximation of the monthly demand charge,  $J_d$ , as defined in (7). Moreover, let

us define  $x := [q, e, n]$  and  $v := [u, \beta]$ , where recall that  $q$  is the power supplied by the utility company,  $e$  is the energy stored in the battery,  $n$  is the number of cycles,  $u$  is the power given to or taken from the battery and  $\beta$  is the battery's degradation rate. Then, Condition I of Theorem 1 holds if we define

$$\psi_{1,N}(x) := p_{\text{off}} \Delta t x_1,$$

$$\begin{aligned} \psi_{1,j}(x, u, \psi_{1,j+1}(y, v)) & := \\ & \begin{cases} p_{\text{on}} \Delta t x_1 + \psi_{1,j+1}(y, v, \psi_{1,j+2}) & \text{if } j \in \Gamma \\ p_{\text{off}} \Delta t x_1 + \psi_{1,j+1}(y, v, \psi_{1,j+2}) & \text{if } j \in \{0, \dots, N\} \setminus \Gamma \end{cases} \quad (17) \end{aligned}$$

$$\begin{aligned} \psi_{2,j}(x, u, \psi_{1,j+1}(y, v)) & := \\ & \begin{cases} p_d^p x_1^p + \psi_{2,j+1}(y, v, \psi_{2,j+2}) & \text{if } j \in \Gamma \\ 0 & \text{if } j \in \{0, \dots, N\} \setminus \Gamma, \end{cases} \quad (18) \end{aligned}$$

where  $N = 30 \times 24/\Delta t$  and  $\Gamma$  is defined as  $\Gamma := \bigcup_{i=1}^{30} \{i \cdot t_{\text{on}}, \dots, i(t_{\text{off}} - 1)\}$ . Condition II follows immediately by using  $\psi_{i,j}$  as defined in (17) and (18). Finally,  $D_1, D_2$  and  $G$  defined in (14), (15) and (16) satisfy Condition III:

$$\frac{\partial G(D_1, D_2)}{\partial D_1} = 1 > 0, \quad \frac{\partial G(D_1, D_2)}{\partial D_2} = \frac{1}{p} D_2^{\frac{1-p}{p}} > 0$$

if we choose  $p$  to be even and

$$\begin{aligned} & \sup \{ |q(1, \dots, t_{\text{on}})|, \dots, |q(30, t_{\text{off}} - 1)| \} \\ & = \sup \{ q(1, \dots, t_{\text{on}}), \dots, q(30, t_{\text{off}} - 1) \} \geq 0. \end{aligned}$$

This corresponds to the typical case where the consumption peak during the on-peak period is positive and is greater than or equal to the maximum power sent back to the grid during the on-peak period. Since all the conditions of Theorem 1 hold, the multi-objective optimization problem

$$\min_{\substack{u(d,k), \beta(d,k) \in \mathbb{R} \\ e(d,k), n(d,k) \in \mathbb{R}}} \left[ D_1 \left( q(1,0), \dots, q \left( 30, \frac{24}{\Delta t} \right) \right), D_2 \left( q(1,0), \dots, q \left( 30, \frac{24}{\Delta t} \right) \right) \right]$$

subject to

$$\begin{aligned} & q(d,k) = q_a(d,k) - q_s(d,k) + u(d,k) \quad \text{for } d \in \Phi_d \text{ and } k \in \Phi_k \\ & e(d, k+1) = e(d,k) + \eta u(d,k) \Delta t \quad \text{for } d \in \Phi_d \text{ and } k \in \Phi_k \\ & e(d,0) = e \left( d-1, \frac{24}{\Delta t} \right) + \eta u \left( d-1, \frac{24}{\Delta t} \right) \Delta t \quad \text{for } d \in \Phi_d \\ & 0 \leq e(d,k) \leq (1 - \beta(d,k)) C_I \quad \text{for } d \in \Phi_d \text{ and } k \in \Phi_k \\ & \beta(d,k) = \alpha n(d,k) + \sum_{k=0}^T \gamma(d,k) \quad \text{for } d \in \Phi_d \text{ and } k \in \Phi_k \\ & n(d, k+1) = n(d,k) + \frac{\eta \Delta t |u(d,k)|}{2c_b(d,k)} \quad \text{for } d \in \Phi_d \text{ and } k \in \Phi_k \\ & \underline{u} \leq u(d,k) \leq \bar{u} \quad \text{for } d \in \Phi_d \text{ and } k \in \Phi_k \\ & e(0,0) = e_0, n(0,0) = 0, \quad (19) \end{aligned}$$

yields an optimal solution to our residential battery optimization problem defined in (8). Various algorithms (e.g.,  $\mathcal{E}$ -constraint [16] and *Envelope* [13]) can be applied to Problem (19) to compute the set of non-dominated solutions. In this paper, we choose to use the *linear scalarization* approach [17]. In this approach, the set of non-dominated solutions to Problem (19) is the parameterized set of solutions to

$$\begin{aligned} & \min D_1(q(1,0), \dots, q(30, 24/\Delta t)) \\ & \quad + \lambda D_2(q(1,0), \dots, q(30, 24/\Delta t)) \quad (20) \end{aligned}$$

TABLE I

SRP'S PRICING PLANS [14]: E-22 (TOU) AND MODIFIED E-27 P (TOU & DEMAND). SUMMER: MAY, JUNE, SEPTEMBER & OCTOBER. SUMMER PEAK: JULY & AUGUST. WINTER: NOVEMBER THROUGH APRIL.

Pricing plan	Off-peak price (\$/kWh)	On-peak price (\$/kWh)	Demand price (\$/kW)
E-22 (Summer)	0.0840	0.3033	0
E-22 (Summer peak)	0.0864	0.3588	0
E-22 (Winter)	0.0758	0.1215	0
E-27 P (Summer)	0.0371	0.0486	14.63
E-27 P (Summer peak)	0.0423	0.0633	17.82
E-27 P (Winter)	0.0390	0.0430	5.68

subject to the constraints of Problem (19), where  $\lambda$  is a positive parameter. Fortunately, the objective function of Problem (20) is time-separable - implying that applying dynamic programming for each  $\lambda > 0$  yields a non-dominated solution to the multi-objective Problem (19).

#### IV. NUMERICAL CASE STUDIES

In this section, we apply our multi-objective approach, described in Section III, to our residential battery optimization problem, defined in Section II, using a wide range of load profiles and two pricing plans. The results are optimal charging/discharging rates as a functions of time, minimum electricity bills, and the amounts which the customers save from optimal energy storage in each scenario.

We consider three load profiles  $q_{a1}$ ,  $q_{a2}$  and  $q_{a3}$  (corresponding to small, medium and large houses) with 5.66 kW, 8.09 kW and 12.14 kW as their maximum values over the summer and 3.89 kW, 5.55 kW and 8.32 kW as their maximum values over the winter. These load profiles were synthesized by averaging the measured load of a group of Salt River Project's (SRP) retail customers in Arizona. As for the electricity pricing, we use SRP's E-22 plan and a modified version of E-27 P plan [14]. The first plan only includes ToU charges, whereas the second plan charges include both ToU and demand charges. The on-peak period for E-22 plan consist of those hours from 4 PM to 7 PM. The on-peak period of plan E-27 P is from 1 PM to 8 PM for the months May to October. From November to April, the on-peak period of plan E-27 P consist of those hours from 5 AM to 9 AM and from 5 PM to 9 PM. The electricity prices for all the seasons are presented in Table I. To synthesize a solar generation profile for each day, we created a random generator which uses a time-varying mean and standard deviation of solar generation for a typical summer day and a time-varying mean and standard deviation for a typical winter day. The means and standard deviations for summer and winter were calculated based on the solar radiation data measured at Equestrian Manor station in Scottsdale, AZ for the entire months July and January. In our scenarios, we consider Tesla's Powerwall battery as the source of energy storage. The two existing models of Powerwall battery possess  $C_l = 7$  kWh and  $C_l = 10$  kWh as storage capacities,  $\bar{u} = 3.3$  kW as max charging rate and  $\eta = 0.92$  as charging efficiency.

##### A. Case Study I: Analysis of The Benefits of Optimal Energy Storage Over a Month

We applied our multi-objective method to the battery optimization problem defined in (8), except that we reduced

TABLE II

TIME OF USE AND DEMAND CHARGES FOR 3 DAYS USING SUMMER PEAK PRICING PLANS E-27 P (TOP) AND E-22 (BOTTOM). (LEFT: WITH BATTERY - RIGHT: WITHOUT BATTERY)

Load profile	ToU charge(\$)	Demand charge(\$)	Monthly bill (\$)	Monthly saving (\$)
$q_{a1}$ (light)	3.30 - 4.03	1.37 - 5.86	46.70 - 98.95	52.25
$q_{a2}$ (medium)	7.45 - 8.20	3.78 - 9.49	112.30 - 176.91	64.61
$q_{a3}$ (heavy)	14.39 - 15.15	9.95 - 16.38	243.43 - 315.04	71.61

Load profile	ToU charge(\$)	Demand charge(\$)	Monthly bill (\$)	Monthly saving (\$)
$q_{a1}$ (light)	2.38 - 11.26	0	23.88 - 112.64	88.76
$q_{a2}$ (medium)	13.59 - 22.47	0	135.94 - 224.71	88.77
$q_{a3}$ (heavy)	32.27 - 41.14	0	322.71 - 411.47	88.76

TABLE III

TOTAL BILLS (FOR 5 YEARS), FINAL CAPACITY OF THE BATTERY, COSTS AND TOTAL SAVINGS FROM BATTERY STORAGE LOAD PROFILES  $q_{a1}$ ,  $q_{a2}$  &  $q_{a3}$ . (LEFT: WITH BATTERY - RIGHT: WITHOUT BATTERY)

Load profile	Total bill (\$)	Final Capacity (kWh)	Cost to Battery (\$)	Total saving (\$)
$q_{a1}$ (light)	1599.13 - 2352.73	9186.1	305.21	448.39
$q_{a2}$ (medium)	3493.93 - 4318.25	9098.3	338.13	486.19
$q_{a3}$ (heavy)	6730.27 - 7657.36	9103.2	336.37	590.72

the 30-day period ( $N = 30$ ) in Problem (8) to 3 Summer peak days in order to simplify the presentation of results. Accordingly, we scaled the demand price in Table I by a factor of  $\frac{1}{10}$ , i.e.,  $p_d = \frac{1}{10} \times 17.82$  \$/kW. In Table II, we have presented the resulting ToU and demand charges for 3 customers with loads  $q_{a1}$ ,  $q_{a2}$  and  $q_{a3}$ , and for pricing plans E-22 and modified E-27 P. Assuming that the three-day simulation is repeated ten times, we calculated the monthly bill by multiplying the sum of 3-day charges by ten. For comparison, we have also included the same results for the case where no battery storage is used. For all the cases associated with modified E-27 P pricing, we solved Problem (20) for 500 values of the weighting parameter  $\lambda$  in (20), using time-step  $\Delta t = 0.5$  hour and  $l_{20}$ -norm approximation for the supremum function in (7). A Matlab implementation of our method calculated the 500 non-dominated solutions in 14.11 minutes using a Core i7 machine with 16 GB of RAM. From Table II we observe that in the presence of demand charges (as in modified E-27 P pricing), the customer with the largest house (heavy load) achieves the greatest saving from optimal energy storage, whereas in the absence of demand charges (E-22 pricing), savings from optimal storage are independent of load. In Figs. 2 and 3, we have shown the resulting optimal charging/discharging rates and optimal energy storage corresponding to the load  $q_{a2}$  for plans E-27 P and E-22 respectively. Fig. 2 shows that in the presence of demand charges, optimal strategy for battery storage involves precise peak shaving (power going into the battery decreases in response to an increase in appliances load) during the on-peak hours. In the absence of demand charges, optimal strategy is straightforward: Apply maximum discharging rate during the on-peak hours (see Fig. 3).

##### B. Case Study II: Long-term Analysis of The Benefits of Optimal Energy Storage

To quantify the long-term benefits of optimal energy storage for residential customers, and assessing battery's degradation under optimal operation, we applied our method

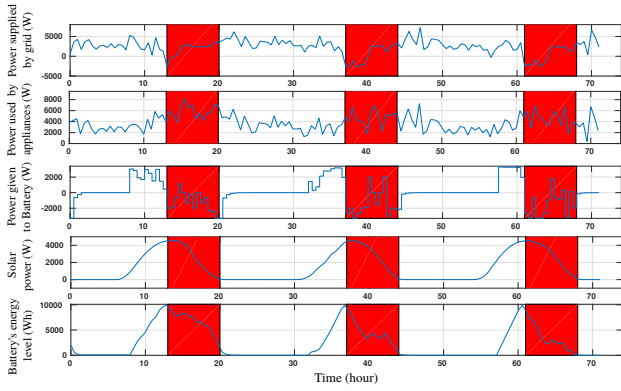


Fig. 2. Optimal charging/discharging rates and energy storage during three Summer peak days for  $q_{2a}$  appliances load and E-27 P pricing plan

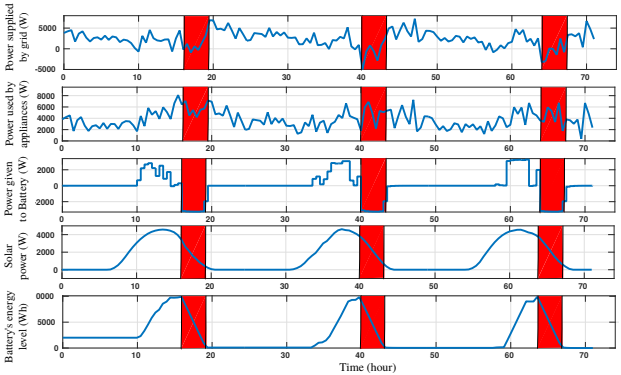


Fig. 3. Optimal charging/discharging rates and energy storage during three Summer peak days for  $q_{2a}$  appliances load and E-22 pricing plan

to Problem (8) for the duration of 5 years. To reduce the computation, we used the appliances load of 3 representative days for each month, assuming that the same load is repeated 10 times during the month. We accounted for the battery's degradation during the entire month by multiplying the degradation rate  $\beta$  (defined in (4)) by a factor of 10. The resulting total bills (for 5 years) corresponding to using  $q_{a1}$ ,  $q_{a2}$  and  $q_{a3}$  as the load profiles and modified E-27 P as the pricing plan are shown in Table III. For comparison, in Table III, we have also included the bills corresponding to no battery storage. Moreover, we have included the cost of operating the battery for 5 years as  $(C_I - C_F)/C_I \times \$3750$  considering that \$3750 is the market price of Tesla's Powerwall battery with  $C_I = 10$  kWh of capacity.  $C_F$  denotes the remaining capacity after 5 years. We have also calculated the total saving from the battery (see Table III) as  $B_b - (C_b + B_n)$ , where  $B_b$  denotes the electricity bill when optimal energy storage is used,  $B_n$  denotes the electricity bill when no energy storage is used, and  $C_b$  denotes the cost of operating the battery. From the table we observe that similar to our analysis in Section IV-A, the customer with the highest load ( $q_{a3}$ ) achieves the greatest benefit from optimal energy storage. Specifically, the customer with load  $q_{a3}$  gains 21% and 32% more benefits relative to the customers with loads  $q_{a2}$  &  $q_{a1}$ .

## V. CONCLUSIONS

We addressed the problem of optimal energy storage for residential customers who are charged for both total energy consumed and peak rate of consumption (demand) during

a month. Because of the presence of demand charges, this problem violates the principle of optimality - a sufficient condition for Hamiltonian-based algorithms and dynamic programming to converge to an optimal solution. We approached this problem by defining a sequence of multi-objective dynamic programs, indexed by  $p$ , whose sets of non-dominated solutions are guaranteed to contain an optimal solution to the original problem as  $p \rightarrow \infty$ . Since the principle of optimality holds for each multi-objective problem in the sequence, standard policy iteration and linear scalarization can be used to find sub-optimal non-dominated solutions for each  $p$ . Our numerical case studies over a wide range of customers' load profiles show that optimal energy storage using Tesla's Powerwall battery and solar generation can reduce the monthly electricity bill by up to 52%, in the presence of demand charges. Moreover, our long-term numerical analysis over the load range, 5.6 kW to 12.1 kW, shows that in the presence of demand charges and battery degradation, a customer with twice the peak load gains 32% higher benefit from optimal energy storage over 5 years.

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