A Parallel Computing Framework for Analysis and Control of Large-scale Systems

Reza Kamyar
Advisor: Matthew Peet
Cybernetic Systems and Controls Laboratory
School for Engineering of Matter, Transport and Energy

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Committee members:
Matthew Peet, chair
Panagiotis Artemiadis, Spring Berman, Georgios Fainekos, Daniel Rivera

We Focus on Two Distinct Topics

Computational focus, **Energy focus**

Topic 1: Application of parallel computing in controls

- Discussing **intractable** problems in control and their real-world applications
- Formulating these problems as optimization problems with a **special structure**
- Designing parallel algorithms capable of exploiting the structure

Topic 2: Optimal thermostat programming in an smart-grid environment

- Determining optimal interior temperature given electricity prices & building parameters
- Benefit to residential customers: minimizing **electricity bills**
- Potential benefit to utility companies: reducing **cost of generation**

Research Goal:

Computational focus, Energy focus

Finding ways to solve **fundamentally difficult** and **large-scale** problems in control.
Problems involving stability and/or control of

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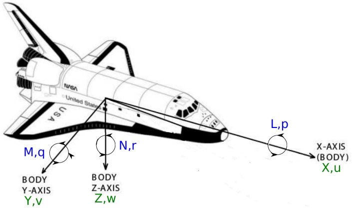
1. System of n linear ODEs with m uncertain parameters ($n > 100$, $m > 10$)

$$\boxed{\dot{x}(t) = A(\alpha)x(t),\ \alpha \in Q \subset \mathbb{R}^m}$$



Application in aerospace:
Linearized equations of symmetric flight:

$$\underbrace{\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} \frac{X_u}{m} & 0 & \frac{X_w}{m} & 0 & \frac{X_q}{m} & 0 \\ 0 & \frac{Y_v}{m} & 0 & \frac{Y_p}{m} & 0 & \frac{Y_r}{m} - U_0 \\ \frac{Z_u}{m} & 0 & \frac{Z_w}{m} & 0 & \frac{Z_q}{m} + U_0 & 0 \\ 0 & \frac{I_{zz}L_v + I_{xz}N_v}{I_{xz}^2 - I_{xx}I_{zz}} & 0 & \frac{-I_{zz}L_p + I_{xz}N_p}{I_{xz}^2 - I_{xx}I_{zz}} & 0 & \frac{-I_{zz}L_r + I_{xz}N_r}{I_{xz}^2 - I_{xx}I_{zz}} \\ \frac{M_u}{I_{yy}} & 0 & \frac{M_w}{I_{yy}} & 0 & \frac{M_q}{I_{yy}} & 0 \\ 0 & \frac{I_{xz}L_v - I_{xx}N_v}{I_{xz}^2 - I_{xx}I_{zz}} & 0 & \frac{I_{xz}L_p - I_{xx}N_p}{I_{xz}^2 - I_{xx}I_{zz}} & 0 & \frac{I_{xz}L_r - I_{xx}N_r}{I_{xz}^2 - I_{xx}I_{zz}} \end{bmatrix}}_{A(X_u, X_w, X_q, Y_v, Y_p, Y_r, Z_u, Z_w, Z_q, L_v, L_p, L_r, M_u, M_w, M_q, N_v, N_p, N_r)} \underbrace{\begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}}_{x(t)}$$

Problem: Find the uncertainty set Q such that for all the aerodynamic coefficients $X_u, X_w, \cdots \in Q$, the aircraft is stable.

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2. Systems of n nonlinear ODEs ($n > 10$)

$$\boxed{\dot{x}(t) = f(x(t))}$$

Application in power systems:
Three-machine, nine-bus power generating system:

$$\dot{\delta}_1(t) = \omega_1(t)$$

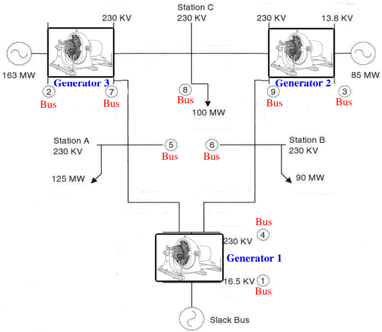
$$\dot{\delta}_2(t) = \omega_2(t)$$

$$\dot{\delta}_3(t) = \omega_3(t)$$

$$\dot{\omega}_1(t) = \frac{1}{m}(d_1\omega_1(t) + P_{m_1} - P_{e_1}(\delta_1(t), \delta_2(t), \delta_3(t)))$$

$$\dot{\omega}_2(t) = \frac{1}{m}(d_2\omega_2(t) + P_{m_2} - P_{e_2}(\delta_1(t), \delta_2(t), \delta_3(t)))$$

$$\dot{\omega}_3(t) = \frac{1}{m}(d_3\omega_3(t) + P_{m_3} - P_{e_3}(\delta_1(t), \delta_2(t), \delta_3(t)))$$



Problem: Find the set of initial phase angles $\delta_i(0)$ and frequencies $\omega_i(0)$ such that $\delta_i(t)$ and $\omega_i(t)$ converge to a stable equilibrium.

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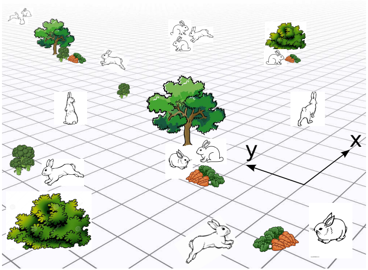
3. PDEs with uncertain parameters

$$\boxed{\frac{\partial}{\partial t}u(x,t) = \alpha_0 u(x,t) + \sum_{i=1}^{m} \alpha_i \frac{\partial^i}{\partial x^i} u(x,t), \quad \alpha \in Q}$$

Application in ecology:

Modelling of population density in a 2D landscape:

$$u_t(x,y,t) = \underbrace{\alpha\big(u_{xx}(x,y,t) + u_{yy}(x,y,t)\big)}_{\text{population diffusion}} + \underbrace{\beta\big(u_x(x,y,t) + u_y(x,y,t)\big)}_{\text{population drift}} + \underbrace{\gamma u(x,y,t)}_{\text{population growth}}$$



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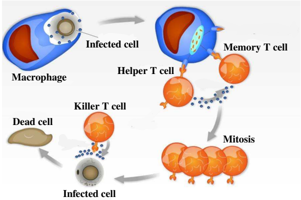
4. Systems of linear ODEs with time-delay

$$\boxed{\dot{x}(t) = \sum_{i=1}^{m} A_i x(t - \tau_i)}$$



Application in immunology:

A linearized model for immune system response:

$$\begin{bmatrix} \dot{T}(t) \\ \dot{T}^*(t) \\ \dot{V}(t) \\ \dot{E}(t) \end{bmatrix} = \begin{bmatrix} -d - kV_{ss} & 0 & -kT_{ss} & 0 \\ kV_{ss} & -\delta - d_x E_{ss} & kT_{ss} - d_x T^*_{ss} & 0 \\ 0 & N\delta & -c & 0 \\ 0 & 0 & 0 & -d_E \end{bmatrix} \begin{bmatrix} T(t) \\ T^*(t) \\ V(t) \\ E(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ p\mathbf{T}^*(\mathbf{t} - \tau) \end{bmatrix}$$

$\mathbf{T}^*(\mathbf{t} - \tau)$ allows for a time delay between the moment of infection and the recognition of the infected cells.

What Are The Computational Challenges?

- **NP-hardness**: Most likely there exists no algorithm which can find exact solutions to these problems in polynomial-time.

  ↳ e.g., Stability analysis of $\dot{x}(t) = A(\alpha)x(t)$ using the converse Lyapunov theory:

  $$P(\alpha) > 0, \;\; A^T(\alpha)P(\alpha) + P(\alpha)A(\alpha) < 0$$

  The question of feasibility of parameter-dependent Lyapunov inequalities is NP-hard.

- **Dimension**: The required memory for the existing algorithms scales exponentially with the dimension of the problem and accuracy of the solutions.

  ↳ Even a "rough" discretization of a 2D PDE can create hundreds of states!

  ↳ e.g., current algebraic geometry techniques (SOS) require 1 TB of memory to verify stability of a nonlinear system with 10 states.

SOS Method To The Rescue!

Polynomial-time asymptotic solutions

- The SOS method defines a sequence of **convex optimization problems** (SOS programs) whose solutions converge to a solution of the intractable problem.
- SOS programs admit **polynomial-time solutions** - complexity $\sim n^{\mathcal{O}(d)}$.
  n : state-space dimension, d : degree of the Lyapunov function

Example: Robust stability

System $\dot{x}(t) = A(\alpha)x(t)$, $\alpha \in [0,1]$ is stable if and only if $\exists\, P(\alpha)$:

$$\boxed{P(\alpha) > 0 \ \text{ and } \ -A(\alpha)^T P(\alpha) - P(\alpha)A(\alpha) > 0 \quad \text{for all} \quad \alpha \in [0,1]}$$

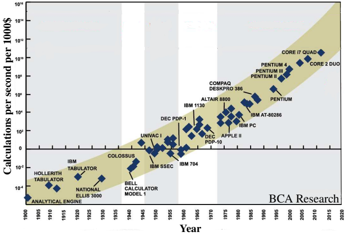
Instead one can solve

$$\boxed{\begin{aligned} P(\alpha) &= S_0(\alpha) + \alpha(1-\alpha)S_1(\alpha) \\ -A(\alpha)^T P(\alpha) - P(\alpha)A(\alpha) &= S_2(\alpha) + \alpha(1-\alpha)S_3(\alpha) \end{aligned}}$$

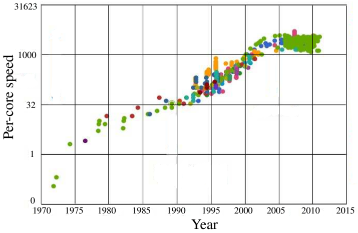
- S_0, S_1, S_2, S_3 are SOS polynomials, i.e., $S_i(\alpha) = \sum_i G_i(\alpha)^2$.

Is Polynomial-Time Good Enough?

- Polynomial-time algorithms have been perceived as the gold standard for what the solution to a control problem should look like.
- However, polynomial-time algorithms are **NOT** always practical!
  - ↳ e.g., computing a Lyapunov function for a 10-state nonlinear system by the SOS algorithm requires 116 DAYS!
- A polynomial-time algorithm is "good" when the ratio of its complexity to the computing power of current computers is reasonably low (technology-dependent).
- The per-core speed of commercial CPUs **has saturated**, while majority of controls algorithms and software can use only a single core.



Moor's law is manifesting in the form of multi-core CPUs.



Single-core processor speed has saturated

Our Contribution: Using Fast-growing Computational Resources For Control

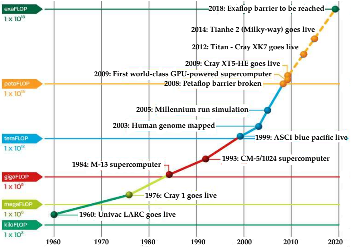
Introducing parallel computation to controls community

- The real problem with computation in control is not the availability of resources, but rather the **lack of algorithms** capable of efficiently utilizing those resources.

- We look for algorithms capable of using those computational resources which have the fastest growth in speed: **cluster-computing, supercomputing**.

- Surprisingly there has been little study on the use of parallel computation for control!

- No surprise! The mathematical machinery for analysis and control is based on two **inherently sequential** algorithms: Linear Programming (LP) & Semi-Definite Programming (SDP)



- How then parallel computing can help? Is it sufficient to focus on parallelizing SDPs with "**special structure**"?

Semi-Definite Programming:

Optimization over the cone of positive semi-definite matrices

$$\boxed{\begin{aligned} \min_{y,Z} \quad & a^T y \\ \text{subject to} \quad & \sum_{i=1}^{K} y_i B_i - C = Z \\ & Z \geq 0 \end{aligned}}$$

- Decision variables: $y \in \mathbb{R}^K$, $Z \in \mathbb{S}^n$ (symmetric matrix)
- SDP elements (given): $B_i, C \in \mathbb{S}^n$ and $a \in \mathbb{R}^K$
- SDPs can be solved efficiently using **interior-point algorithms**.

Interior-point algorithms solve SDPs in TWO steps:

1. Reducing the SDP to a sequence of optimization programs with only equality constraints

Dual SDP

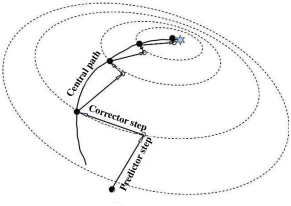
$$\begin{aligned} \min_{y,Z} \ & a^T y \\ \text{subject to} \ & \sum_{i=1}^{K} B_i y_i - C = Z \\ & Z \geq 0 \end{aligned}$$

Approximation using barrier function

$$\begin{aligned} \min_{y,Z} \ & a^T y - \mu \log(\det(Z)) \\ \text{subject to} \ & \sum_{i=1}^{K} B_i y_i - C = Z \end{aligned}$$

2. Applying a descent algorithm, e.g., Newton's algorithm, to solve the equality constrained problems

$$\begin{aligned} y^{k+1} &= y^k + t\,\Delta y^k \\ Z^{k+1} &= Z^k + t\,\Delta Z^k \\ X^{k+1} &= X^k + t\,\Delta X^k \end{aligned}$$



- $\Delta y^k, \Delta Z^k, \Delta X^k$ are the step directions.
- Calculating the step directions is the most computationally expensive part

What If The SDP Elements Have a Special Structure?

Block-diagonality is preserved through iterations

Assumption: The SDP elements B_i and C are **block-diagonal** matrices.

Primal step: $$\Delta X^k = -X^k + {Z^k}^{-1}\left(-\sum_{i=1}^{K} B_i y_i^k + Z^k + C\right)\sum_{i=1}^{K} B_i \Delta y_i^k X^k$$

Observations:

- If X^0 and Z^0 are **block-diagonal**, then ΔX^k and ΔZ^k are **block-diagonal** $\forall k$.
- Then X^k and Z^k are also **block-diagonal** for all k because
$$X^{k+1} = X^k + t\Delta X^k \qquad Z^{k+1} = Z^k + t\Delta Z^k.$$
- We decentralize the computation of step directions $\Delta X, \Delta Z$ by assigning each block to a processor.

Can Stability/Control Problems Reduce To Block-diagonal SDPs?

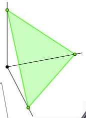
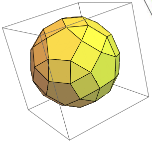
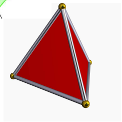
Alternatives to SOS algorithm

- Unfortunately SOS algorithm does NOT yield block-diagonal SDPs.

  Example: Is $f(x) = 4x^4 + 4x^3y - 7x^2y^2 - 2xy^3 + 10y^4 \geq 0$?

  $$\text{If } \exists M := \begin{bmatrix} M_1 & M_2 & M_3 \\ M_2 & M_3 & M_4 \\ M_3 & M_4 & M_5 \end{bmatrix} \geq 0 \text{ such that } f = \begin{bmatrix} x^2 \\ xy \\ y^2 \end{bmatrix}^T M \begin{bmatrix} x^2 \\ xy \\ y^2 \end{bmatrix} \Rightarrow f \text{ is SOS}$$

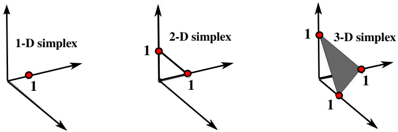
- We identified alternatives to SOS - Theorems which reformulate polynomial positivity (e.g., $V > 0, \dot{V} < 0$) as feasibility of **block-diagonal SDPs** and LPs:
  - **Polya's Theorem** (positivity over the standard unit simplex)
  - **Bernstein's Theorem** (positivity over simplex)
  - **Handelman's Theorem** (positivity over polytopes)

Polya's Theorem

A test for non-negativity over the standard simplex

Example: Is $p(x,y) = 2x^4 - 0.11x + y^3 \geq 0$?



Step 1) **Homogenizing $p(x,y)$:**

$$\tilde{p}(x,y) = 2x^4 - 0.11x(x+y)^3 + y^3(x+y)$$

Step 2) **Polya's iterations on $\tilde{p}(x)$:**

Multiply $\tilde{p}(x,y)$ by $(x+y)$ until all the coefficients are positive.

Iteration #1:

$$(x+y)\tilde{p}(x,y) = 1.89x^5 + 1.56x^4y - 0.66x^3y^2 + 0.56x^2y^3 + 1.89xy^4 + y^5$$

Iteration #2:

$$(x+y)^2\tilde{p}(x,y) = 1.89x^6 + 3.45x^5y + 0.9x^4y^2 - 0.1x^3y^3 + 2.45x^2y^4 + 2.89xy^5 + y^6$$

Iteration #3:

$$(x+y)^3\tilde{p}(x,y) = 1.89x^7 + 5.34x^6y + 4.35x^5y^2 + 0.8x^4y^3 + 2.35x^3y^4 + 5.34x^2y^5 + 3.89xy^6 + y^7$$

- Recall that $\dot{x}(t) = A(\alpha)x(t)$, $\alpha \in \Delta$ is stable if and only if $\exists\, P(\alpha)$:

$$\boxed{P(\alpha) > 0 \;\text{ and }\; -A(\alpha)^T P(\alpha) - P(\alpha)A(\alpha) > 0 \;\text{ for all }\; \alpha \in \Delta}$$

- Let $P(\alpha)$ be of the form

$$P(\alpha) = P_1\alpha_1^2 + P_2\alpha_1\alpha_2 + P_3\alpha_2^2 \qquad (P_i \in \mathbb{S}^m \text{ are unknown})$$

Then by calculating the coefficients of $(\alpha_1 + \alpha_2)P(\alpha)$ as

$$(\alpha_1 + \alpha_2)\, P(\alpha) = P_1\, {\alpha_1}^3 + (P_1 + P_2){\alpha_1}^2\, \alpha_2 + (P_2 + P_3)\, \alpha_1\, {\alpha_2}^2 + P_3\, {\alpha_2}^3,$$

positive definiteness of $P(\alpha)$ is guaranteed if $\exists P_1, P_2, P_3$ such that

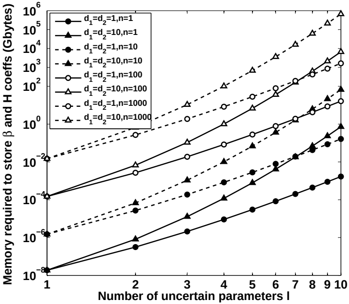
$$\begin{bmatrix} P_1 & 0 & 0 & 0 \\ 0 & P_1 + P_2 & 0 & 0 \\ 0 & 0 & P_2 + P_3 & 0 \\ 0 & 0 & 0 & P_3 \end{bmatrix} > 0.$$

- Similarly, we can apply Polya's theorem to $A(\alpha)^T P(\alpha) + P(\alpha)A(\alpha) \leq 0$.

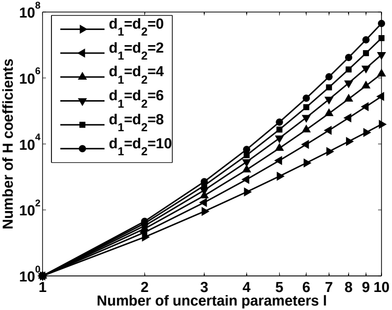
The Resulting SDPs Are Large!

The required memory for setup and solving the SDPs is beyond desktop/shared-memory computers

Memory required for storing the SDP



Number of monomials in $(\sum \alpha_i)^d A^T P + PA$



- Number of SDP **variables**: $\boxed{\sim n^2\, l^{d_p}}$

- Number of SDP **constraints**: $\boxed{\sim n\, l^{d_p + d_a + d}}$

Recall:

n : state-space dimension l : number of uncertain parameters

d : number of Polya's iterations d_p, d_a : degrees of $P(\alpha)$ and $A(\alpha)$

We Designed And Implemented TWO Parallel Algorithms: Setup & Solver

- **Parallel setup algorithm:**
  1. Distributes monomials of $P(\alpha)$ and $A(\alpha)$ among processors, evenly.
  2. Each processor applies Polya's iteration to its monomials:

$$Q_1 = \Big(\sum_i \alpha_i\Big) P(\alpha) \quad \text{and} \quad Q_2 = \Big(\sum_i \alpha_i\Big) A^T(\alpha) P(\alpha) + P(\alpha) A(\alpha)$$

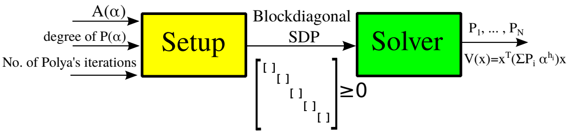
  3. Redistributes the monomials of Q_1 and Q_2 among processors, evenly (Communication)

- **Parallel SDP solver:**
  - Recall that the step directions ΔX and ΔZ are **block-diagonal**.

$$\Delta X = \operatorname{diag}\{\Delta X_1, \cdots, \Delta X_M\} \quad \Delta Z = \operatorname{diag}\{\Delta Z_1, \cdots, \Delta Z_M\}$$

  - Having N Processors, each processor computes at least $\texttt{floor}(\frac{M}{N})$ blocks and updates

$$X_i = X_i + t\Delta X_i, \qquad Z_i = Z_i + t\Delta Z_i \qquad \text{for } i = 1, \cdots, \texttt{floor}\left(\frac{M}{N}\right)$$



Per-Core Complexity of The Algorithms Is $\mathcal{O}(n^7)$

Assumptions:

1. Having sufficiently large number of processors ($\geq$ number of blocks)
2. Number of states, $n \gg$ number of uncertain parameters, l

Then

1. Our algorithms solve robust stability problem

$$A^T(\alpha)P(\alpha) + P(\alpha)A(\alpha) < 0 \quad \alpha \in \Delta^l$$

   with the **same per-core cost** $\mathcal{O}(n^7)$ as required for solving the stability problem

$$A^T P + PA < 0.$$

2. Increasing accuracy (performing Polya's iterations) does **NOT** add any per-core computation and communication.

Theoretically Our Algorithms Achieve Linear Speed-up

Speed-up: The ratio of the execution time using one core to the execution time using $N \geq 1$ cores.

- Potential speed-up is calculated as

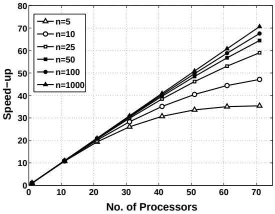
$$\boxed{SP = \frac{N}{D + NC}}$$

D: decentralized computation $\qquad$ C: centralized computation

- For sufficiently large number of processors, we have shown

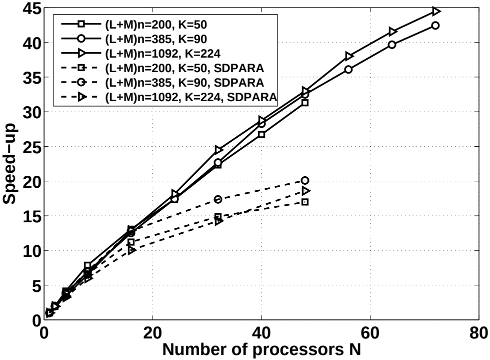
$$\lim_{n\to\infty} D(n) = 1 \text{ and } \lim_{n\to\infty} C(n) = 0.$$

$$\Rightarrow \lim_{n\to\infty} SP(n) = \lim_{n\to\infty} \frac{N}{D(n) + NC(n)} = N \quad \textbf{(Linear speed-up)}$$



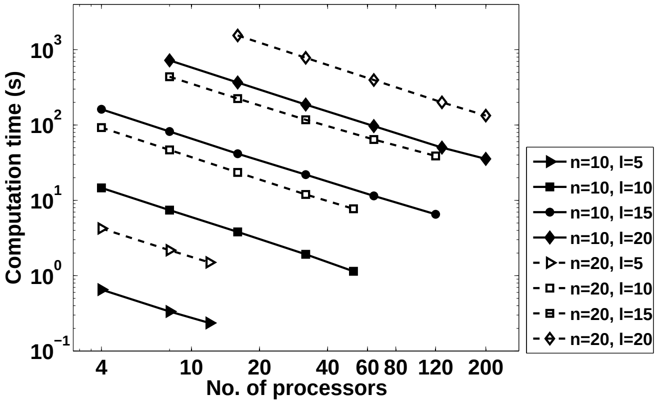
Linear Experimental Speed-up of Our Parallel SDP Solver

Our parallel SDP solver outperforms the general purpose parallel SDP solver, SDPARA, in terms of speed-up



Linear Experimental Speed-up of Our Parallel Set-up Algorithm

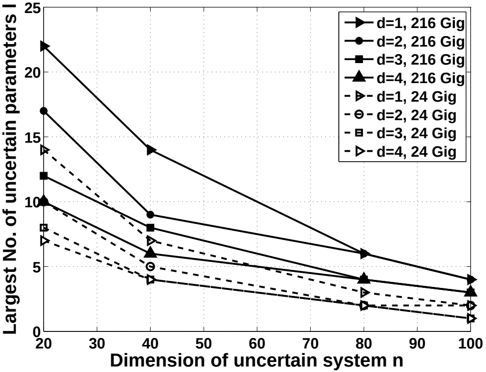
Computation time of the **set-up algorithm** scales **log-linearly** with number of cores



Executed on IBM's Blue-Gene supercomputer at Argonne National Laboratory

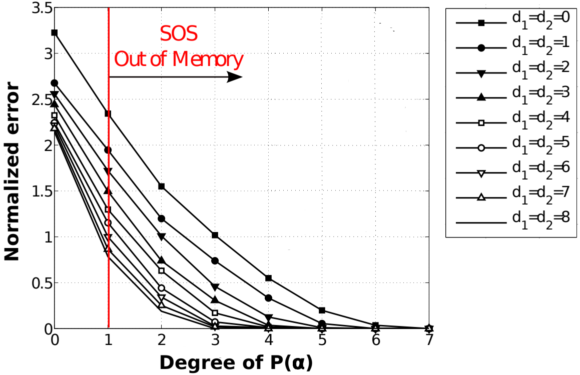
How Big A Problem Can The Algorithms Solve?

The proposed decentralized algorithms can solve problems with **100+** state-space dimension



Executed on one and nine nodes of Karlin cluster computer with 24GByte/node RAM

Conservatism Reduces As Degree of P & No. of Polya's Iterations Increase



Error of algorithm's approximation for the largest r such that $\dot{x}(t) = A(\alpha)x(t)$ is stable $\forall \alpha \in \Delta_r$

$$\Delta_r := \{\alpha \in \mathbb{R}^l : \sum_{i=1}^{l} \alpha_i = r,\ \alpha_i \geq 0\}$$

- SOS algorithm **runs out of memory** for $d_p \geq 2$

Summary of Contributions

Computational focus, Energy focus

- Designed a parallel SDP solver for block-diagonal SDPs (ACC 2012)

- Designed a parallel setup algorithm to apply Polya's theorem to robust stability over the simplex (TAC 2013)

$$\Delta^n := \{x \in \mathbb{R}^n : \sum_{i=1}^{n} x_i = 1, x_i \geq 0\}$$

- Extending Polya's theorem for robust stability over hypercubes (CDC 2012)

$$\Phi_r^n := \{x \in \mathbb{R}^n : |x_i| \leq r_i,\, i = 1, \cdots, n\}$$

- Extension to nonlinear local stability/region of attraction estimation inside hypercubes (CDC 2013)

- Extension to stability over arbitrary polytopes using Handelman's theorem (CDC 2014)

$$\Gamma^K := \{x \in \mathbb{R}^n : w_i^T x + u_i \geq 0,\, i = 1, \cdots, K\}$$

- A survey on alternatives to SOS (Polya, Handelman, Bernstein, Blossoms, $\cdots$) (DCDS 2015)

- Generalizing our parallel set-up algorithm to apply Polya's theorem to **arbitrary** parameter-dependent inequalities of the form:

$$\sum_{i=1}^{N}\Big(A_i(\alpha)X(\alpha)B_i(\alpha)+B_i^T(\alpha)X(\alpha)A_i^T(\alpha)+R_i(\alpha)\Big)<-\gamma I \quad \text{for all} \quad \alpha\in Q,$$

- Parallel algorithm for **Optimal Control**:

$$\begin{aligned} J^* := \min_{u_k\in U} \; & \sum_{k=0}^{\infty}\beta^k g(x_k,u_k) & \\ \text{subject to} \;\; & x_{k+1}=f(x_k,u_k) & \text{for } k=1,2,3,\cdots \\ & x_k\in X,\; x_0=z & \text{for } k=1,2,3,\cdots \end{aligned}$$

By searching for polynomial value functions V which satisfy Bellman's formula:

$$\boxed{V(z)=\inf_{v\in U}\{g(z,v)+\beta\, V(f(z,v))\}\quad \forall z\in X.}$$

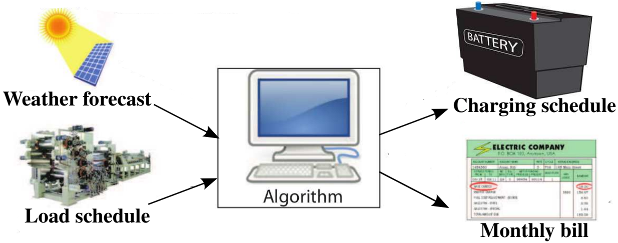
Then $V(z)=J^*$.

Transition To Our Second Topic: Optimal Thermostat Programming

Computational focus, **Energy focus**

- Computing **optimal response** of residential customers to electricity prices
  - Quantifying the benefits of using **energy storage** and **solar** by the customers
  - Minimizing the electricity bill by designing **optimal thermostats** for HVAC systems

- Economical **implications** for power companies
  - Optimal electricity pricing for minimizing cost of generating electricity
  - Optimal unit scheduling



Power Companies Pay For Fuel And Building/Maintenance of Generators

A simplified model for cost of generating electricity is a combination of

1. **Cost of fuel** required to generate the total energy (kWh) consumed by users

   A common model is: $$\boxed{\text{cost of fuel} = a \int q(t)dt}$$

   - $q(t)$ (kW): power consumed by users
   - a ($/kWh): cost of fuel required to produce the next kWh

2. **Cost of building & maintaining generators** to accommodate for the maximum total power (kW) consumed by users

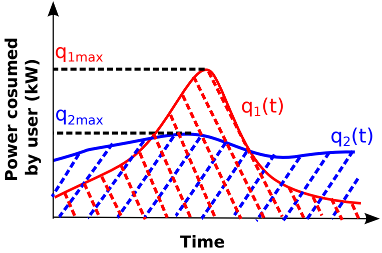
   A simple model can be:

   $$\boxed{\text{Cost of building \& maintaining generators} = b \sup_{t \in \text{on-peak}} q(t)}$$

   - b ($/kW): cost of installing the next kW of generating capacity

Current Pricing Strategies Do Not Charge For Peak Consumption

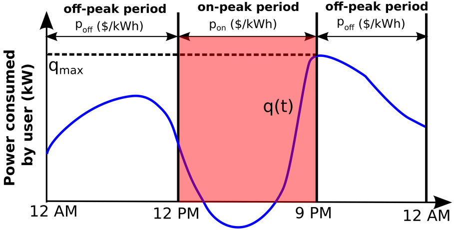
- Most power companies use **flat** or **Time-of-Use** (ToU) pricing
  - ↳ **Flat pricing:** Charges are independent of when energy is used



$$\int q_1(t)dt \times \frac{\text{price}}{kWh} = \int q_2(t)dt \times \frac{\text{price}}{kWh}$$

Electricity bills independent of $q_{1\,\max}$ & $q_{2\,\max}$

↳ **ToU pricing**: Does not explicitly charge for max power used



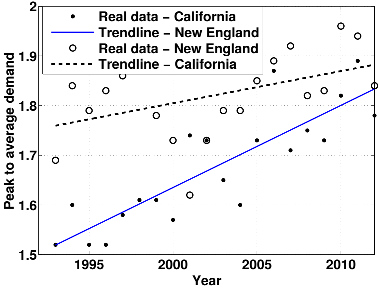
$$\text{Elect. Bill} = p_{\text{off}} \int_{\text{off-peak}} q(t)dt + p_{\text{on}} \int_{\text{on-peak}} q(t)dt$$

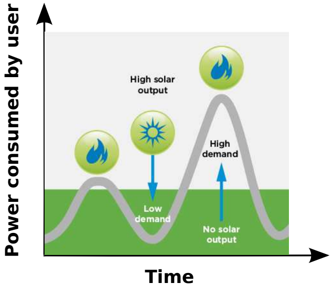
Large peak does not necessarily result in a higher monthly bill!

Current Pricing Strategies Are Problematic For Power Companies

- **Fact 1:** The ratio of **maximum power** used per year to **average power** used per year is setting records in the US!

  ↳Partially due to increasing **integration of renewables**, e.g., solar.



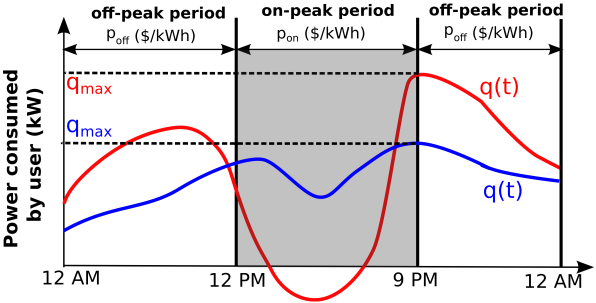


- **Fact 2:** Integration of renewables does **NOT** affect maximum power consumption, but reduces the total power sold by power companies ⇒ **revenue decreases**

- **Consequence:** Power companies won't have enough revenue to supply for electricity without raising the prices

Demand Charge: A Solution To The Revenue Problem

- **Demand charge**: A monthly charge proportional to the maximum power consumed by the user during the on-peak hours of a month
- A combination of off-peak, on-peak and demand charges can differentiate between "**good**" and "**bad**" user behavior



$$\text{Electricity Bill} = \underbrace{p_{\text{off}} \int_{t \in \text{off-peak}} q(t)dt}_{\text{on-peak period charge}} + \underbrace{p_{\text{on}} \int_{t \in \text{on-peak}} q(t)dt}_{\text{off-peak period charge}} + \underbrace{p_d \sup_{t \in \text{on-peak}} q(t)}_{\text{demand charge}}$$

How Can Power Companies Optimize Their Prices?

Power companies can solve the following optimization problem:

- **Objective:** minimize the cost of generating electricity

$$\min_{p_{\text{on}},p_{\text{off}},p_d} \left(\underbrace{\int_{t=0}^{t=24} (a\, g(t)^2 + b\, g(t))dt}_{\text{fuel cost}} + \underbrace{c \sup_{t \in \text{ on-peak period}} g(t)}_{\text{cost of building generators}} \right)$$

  - $g(t)$: power (kW) generated at time t
  - a, b (\$/kWh): fuel cost coefficients
  - c (\$/kW): cost of installing the next kW of production capacity

- **Constraint:**
  - Equality of **generation**, $g(t)$, and **consumed power**, $q_{\text{user}}(t)$:

$$\boxed{g(t) = q_{\text{user}}(t, p_{\text{off}}, p_{\text{on}}, p_d) \quad \forall t}$$

- **Variables:** on-peak, off-peak and demand prices: $p_{\text{on}}, p_{\text{off}}, p_d$

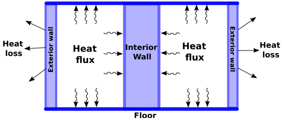
Power Companies Need A Model For User Behavior

- To optimize electricity prices, we need a **model** for **users' power consumption**
  ↳ Model should Predict how much electricity would a **rational** user consume, given the prices

- **Question**: How can a rational user reduce his electricity bill?
  ↳ One way is to reduce HVAC load by using **Energy storage**

  1. Energy storage in residential **batteries** allows users to shift peaks from high-demand hours to other hours

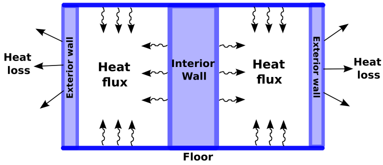
  2. Using walls/floors as **thermal energy storage**: A free alternative to batteries

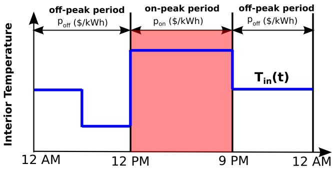


Power Companies Need A Model For User Behavior

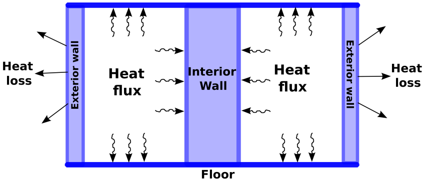
Precooling exploits thermal energy storage in walls to shift loads:

- Cool down walls/floors when electricity is cheap





- Cold walls will reduce the load on HVAC during on-peak hours - thus reducing the electricity bill



How Do Thermostat Settings Affect Energy Consumption?

Power consumed by user is a combination of heat **loss to outside** and heat **given to/taken from interior walls**

$$\boxed{q_{\text{user}}(t) = q_{\text{loss}}(t) + q_{\text{wall}}(t) \quad \forall k}$$

- Heat loss $q_{\text{loss}}(t)$ is modeled by a **linear heat sink** and can be controlled by interior temperature T_{in}:

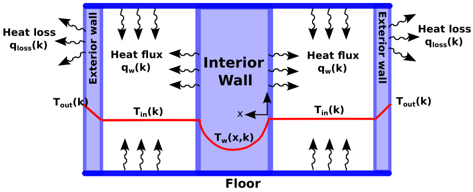
$$q_{\text{loss}}(t) = \frac{T_{\text{out}}(t) - T_{\text{in}}(t)}{R_w}$$

T_{out} : Outside temperature $\quad R_w$: thermal resistance

- Heat thru walls $q_{\text{wall}}(k)$ is modeled by the **Heat equation** (PDE):

$$\frac{\partial T_w(t,x)}{\partial t} = \alpha \frac{\partial^2 T_w(t,x)}{\partial x^2}$$

$$q_{\text{wall}}(k) = 2C_w \frac{\partial T_w}{\partial x}(t,0)$$



How Do Rational Users Minimize Their Electricity Bill Including Demand Charges?

User can solve a discrete-time **thermostat programming** problem with

- **Objective:** minimize the electricity bill

$$\min_{T_{\text{in}}(k)} \left(\underbrace{30\, p_{\text{off}} \sum_{k \in I_{\text{off}}} q_{\text{user}}(k)}_{\text{OFF-peak period charge}} + \underbrace{30\, p_{\text{on}} \sum_{k \in I_{\text{on}}} q_{\text{user}}(k)}_{\text{ON-peak period charge}} + \underbrace{p_d \sup_{k \in I_{\text{on}}} q_{\text{user}}(k)}_{\text{demand charge}} \right)$$

- **Constraints:**
  1. Interior temperature with a certain bound:
  $$T_{\min} \le T_{\text{in}}(k) \le T_{\max} \quad \forall k$$
  2. Energy conservation:
  $$q_{\text{user}}(k) = q_{\text{loss}}(T_{\text{in}}(k), T_e(k)) + q_{\text{wall}}(T_w(x,k)) \quad \forall k$$
  3. Discretized heat dynamics: $T_w(k+1) = A\, T_w(k) + B\, T_{\text{in}}(k)$
- **Variables:** Interior temperature $T_{\text{in}}(k)$ over time

A Reformulation of User's Problem Can Be Solved By Dynamic Programming

- We first reformulate the user's problem

$$\begin{array}{lll} \min\limits_{T_{\text{in}}(k)} & 30\, p_{\text{off}} \sum\limits_{k \in I_{\text{off}}} q(k) + 30\, p_{\text{on}} \sum\limits_{k \in I_{\text{on}}} q(k) + \boxed{p_d \sup\limits_{k \in I_{\text{on}}} q(k)} & \\ \text{subject to} & q(k) = q_{\text{loss}}(T_{\text{in}}, T_{\text{out}}) + q_{\text{w}}(T_w) & \forall k \\ & T_w(k+1) = f(T_w(k), T_{\text{in}}) & \forall k \\ & T_{\min} \leq T_{\text{in}}(k) \leq T_{\max} & \forall k \end{array}$$

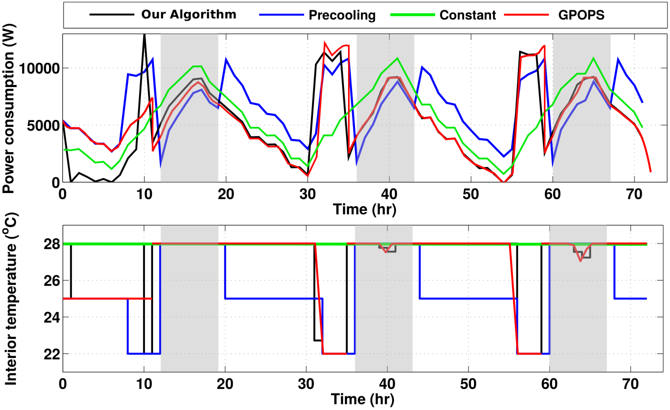
as

$$\begin{array}{lll} \min\limits_{T_{\text{in}}(k), \gamma \in \mathbb{R}} & 30\, p_{\text{off}} \sum\limits_{k \in I_{\text{off}}} q(k) + 30\, p_{\text{on}} \sum\limits_{k \in I_{\text{on}}} q(k) + p_d \gamma & \\ \text{subject to} & \boxed{q(k) \leq \gamma} & \forall k \in I_{\text{on}} \\ & q(k) = q_{\text{loss}}(T_{\text{in}}, T_{\text{out}}) + q_{\text{w}}(T_w) & \forall k \\ & T_w(k+1) = f(T_w(k), T_{\text{in}}) & \forall k \\ & T_{\min} \leq T_{\text{in}}(k) \leq T_{\max} & \forall k \end{array}$$

- For fixed γ, the reformulated problem can be solved by **Dynamic Programming**.
- γ is a scalar, so we use **bisection** over γ.

Our Algorithm Can Reduce electricity Bills By Up To 25% (average 9.2%)

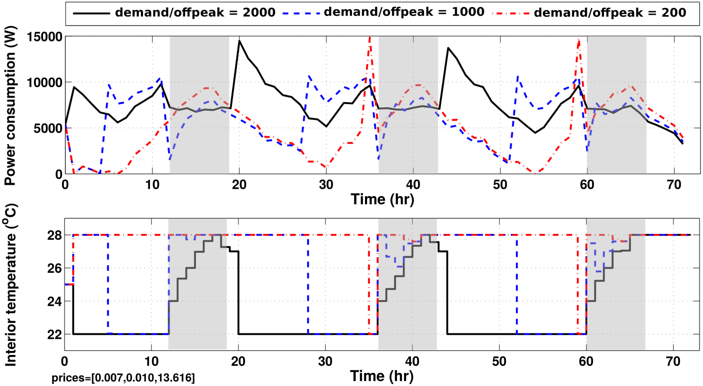
User's consumption and interior temperature using prices from Arizona Public Service (APS)



Temperature setting	Our algorithm	GPOPS	Pre-cooling	Constant
Monthly bill	**365.8$**	370.3$	392.3$	394.2$

Increasing $\frac{p_d}{p_{\text{off}}}$ Helps Reducing Maximum Consumption during on-peak

- Weight of **demand** price relative to **on-peak** & **off-peak** prices affects maximum consumption during on-peak hours

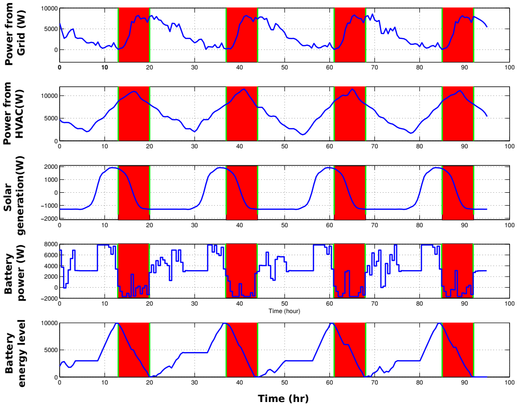


Summary of Contributions on Thermostat Programming/Electricity Pricing

- Defined a new model for optimal behavior of a customer who minimizes his electricity bill based on given prices (ACC 2015)
  - ↳ Including thermal energy storage using the heat equation
  - ↳ including monthly demand charges
- Used our model to define a framework for optimization of electricity prices for rational users (submitted to IEEE Transactions on Power Systems)
  - ↳ Minimizing the cost to the power company
  - ↳ Considering integration of solar power
- A Multi-objective Approach To Optimal Battery Storage In The Presence of Demand Charges (Under preparation for IBO Conference, 2016)

Our Ongoing Research On Storage: Optimal Programming of Batteries

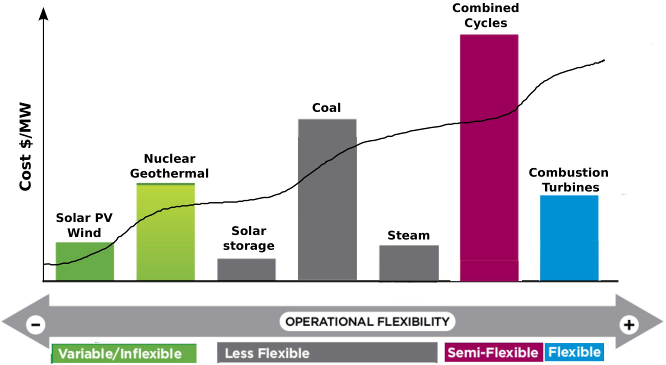
- Incorporating **batteries**, such as Tesla's Powerwall & Tesla's Powerpack in our user's models and utility model



- Including **stochasticity** due to weather temperature and solar radiation in our customer's model - minimizing $E_\omega\{\sup_u g(t, u, \omega)\}$.

Our Ongoing Research: Benefits of Battery Storage To Power Companies

- Optimal **battery storage** & **unit scheduling** to minimize generation costs
  - ↳ Fuel cost of various types of generating units
  - ↳ Unit commitment: Cost for bringing each generating unit online
  - ↳ Arbitrage: Selling/buying from electricity spot market
  - ↳ Spinning reserve and frequency regulation costs



Conclusions & Achievements

Topic 1: Application of parallel computing in controls

- Developed a parallel optimization framework using **Polya's & Handelman's** theorems for robust stability analysis over various geometries.
- Our algorithms achieve **near-linear** theoretical and experimental speed-up.
- Out algorithms enable robust stability analysis of systems **3 times** larger than ANY other algorithm (100+ states, tens of parameters).

Topic 2: Optimal thermostat programming in an smart-grid environment

- Developed a model for rational customers who **exploit storage** to minimize their monthly bill
- Designed an algorithm for **optimal thermostat programming**, capable of reducing monthly bills by up to 25%
- Proposed **optimal combinations** of on-peak, off-peak, demand prices which reduce both peak consumption and generation costs
- Quantified the effects of **solar integration** on customers behavior and generation costs

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