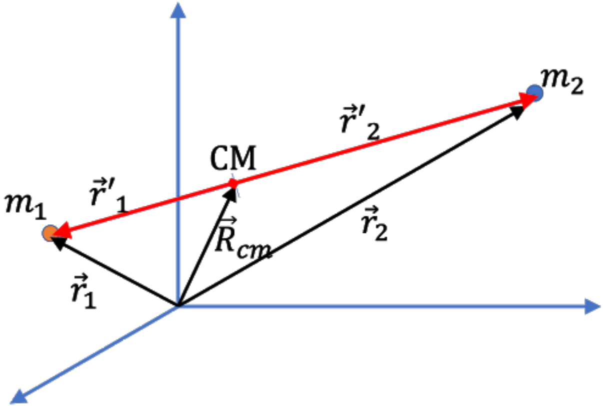
Spacecraft Dynamics and Control

Matthew M. Peet

Lecture 1: In the Beginning

Our Model: The 2-Body Problem

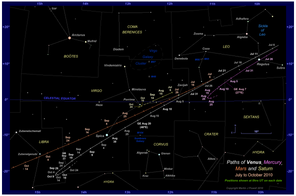
Modeling the system is 90% of the problem.

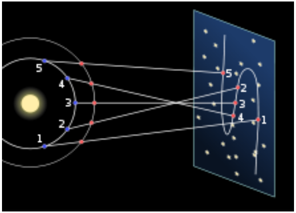


But where did this model come from?

- That is the longest backstory ever.

Celestial Models Are Built to Explain Observations



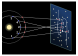


Observations to explain:

- Stars rotate about a fixed point (north star)
- Lunar and Solar Eclipses
- Planets migrate in a line among the stars
  - Generally forward motion, but occasionally retrograde (Mars).
- The Moon
- Lack of Stellar Parallax
- Different Stars in North and South
- Seasonal Changes in Daylight
- Changes in Planetary Brightness
- Ocean Tides

Celestial Models Are Built to Explain Observations





Observations to explain:

- Stars rotate about a fixed point (north star)
- Lunar and Solar Eclipses
- Planets migrate in a line among the stars
  - Generally forward motion, but occasionally retrograde (Mars).
- The Moon
- Lack of Stellar Parallax
- Different Stars in North and South
- Seasonal Changes in Daylight
- Changes in Planetary Brightness
- Ocean Tides

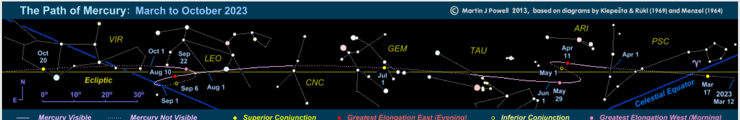
Image credits: None of the images in any of these lectures are created by me. They are all pulled from the internet. Whenever original illustrator/author information is available, I will add credit. None of these images should be taken for commercial use.
`https://vimeo.com/18755092`
`https://www.dreamstime.com/stock-photo-girl-looking-stars-girl-pointing-shooting-star-image97258129`

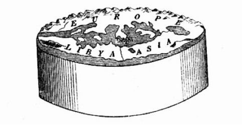
Another Observation: Spring and Summer are collectively 8 days longer than fall and winter in the northern hemisphere.

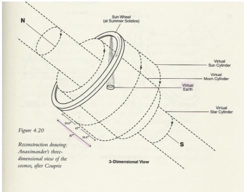
Never underestimate the importance of models (inductive reasoning).

- We have mathematics, because it is useful for modeling.
- In the beginning, there was no math, so we used mechanical devices for our models
  - Gears (Anaximander)
  - Then circles (Ptolemy)
  - Then forces (Gallileo)
  - Then ellipses (Kepler)
  - Then differential equations (Newton)
- Today, differential equations and mathematical models are the basis of all technology, from aircraft, to rockets, to artificial intelligence
- Machine learning is the part of AI for creating models.

Models must be predictive

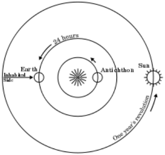


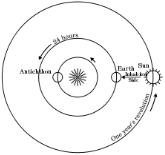




Anaximander's Model

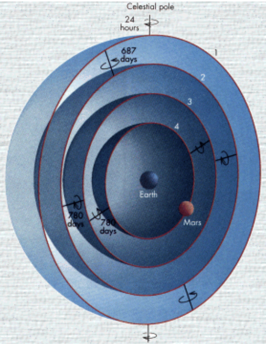
- Cylindrical Floating Earth
- Invented the concept of Space





Philolaus' Model

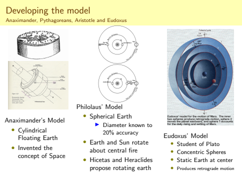
- Spherical Earth
  - Diameter known to 20% accuracy
- Earth and Sun rotate about central fire
- Hicetas and Heraclides propose rotating earth



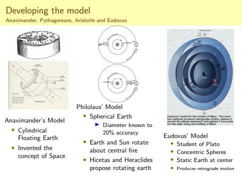
Eudoxus' model for the motion of Mars. The inner two spheres produce retrograde motion, sphere 2 moves the planet eastward, and sphere 1 accounts for the daily rising and setting of Mars.

Eudoxus' Model

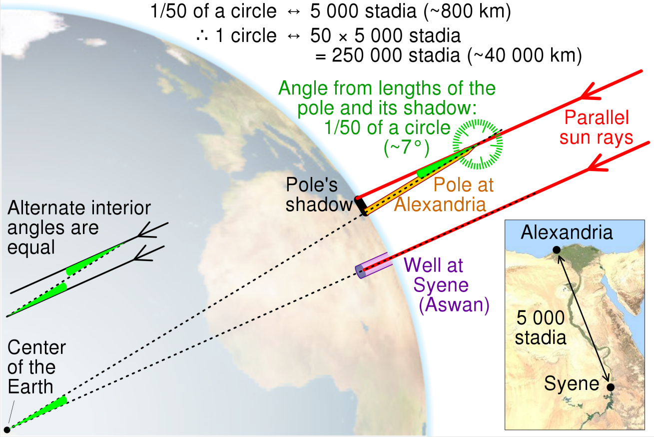
- Student of Plato
- Concentric Spheres
- Static Earth at center
- Produces retrograde motion



- Anaximander (600 BC, Miletus) was the first to conceive a *mechanical* model of the world, projecting models and constructions of man onto cosmology (contrast to egg model of e.g. Zhang Heng - c. 100AD). In his model, the Earth floats very still in the centre of the infinite, not supported by anything (Geocentric). Its shape is that of a cylinder with a height one-third of its diameter. The flat top forms the inhabited world, which is surrounded by a circular oceanic mass. Anaximander's hypothesis presumes that the Earth floats without falling and does not need to be resting on something. Explains seasonal changes in daylight. Recall Greece was in a dark age until 800 BC with no written language.
- Compare this with Homer: "Okeanos, the personified body of water surrounding the circular surface of the Earth" on shield of Achilles. Common to Egyptians, Mesopotamians, Israelites, Norse, Germanic, Chinese (square earth, round heaven), Indian (Vedic text describe a stack of disks - heaven and earth). Many of these traditions continued into the 1500s-1600s.
- The Pythagoreans and Philolaus (developed pythagorean theorem), in particular, in the 5th century BC expand on Anaximander's model of infinite space, but make earth spherical, place the center at an unseen "fire" (dark matter?), and create a dark counter-earth to explain eclipses of the moon. The motion of the earth about the central fire is daily, not yearly, however, and hence this is not a true rotating earth model for explaining day/night. Rotation is daily so that we never see central fire. Hicetas and Heraclides Ponticus (390-310 BC), however, clearly refer to an earth that rotates on its axis.
- The spherical earth theory is supported by the fact that travelers to the south observe constellations rise higher above the horizon (impossible in flat earth model) and the lunar eclipse is in the form of a circle. Spherical earth models were universal in Greece by the end of the 5th century BC.
- Eudoxus of Cnidus (390-337 BC), together with Plato and Aristotle, proposed the Geocentric model based on celestial spheres. Problems with this model include: 1. the retrograde motions all have the same shape. 2. The model works well for some planets, but not others. 3. The model does not account for observed changes in brightness of the planets over time.



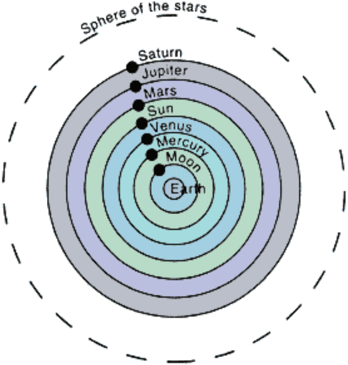
- Eratosthenes (276 - 194 BC from Cyrene, a greek colony in Libya (Egypt)), observed that in Syene the Sun was directly overhead at summer solstice whereas in Alexandria it cast a shadow with a known angle. He estimated the circumference at 250,000 stades. Although the length of a 'stade' is not precisely known, it is supposed that his estimate accurate to between 2% and 20% (1ER= 40,008 km). Starved himself to death at age 82 after becoming blind.



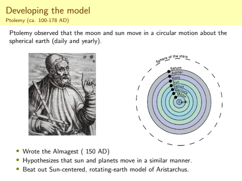
Developing the model

Ptolemy (ca. 100-178 AD)

Ptolemy observed that the moon and sun move in a circular motion about the spherical earth (daily and yearly).

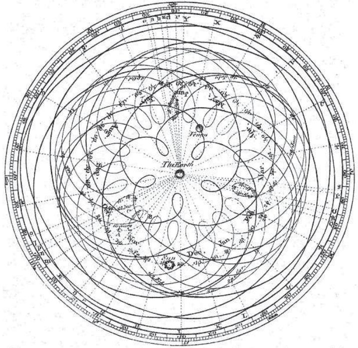


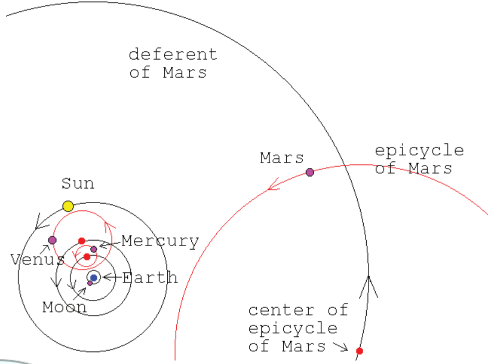
- Wrote the Almagest (150 AD)
- Hypothesizes that sun and planets move in a similar manner.
- Beat out Sun-centered, rotating-earth model of Aristarchus.



- Ptolemy expands the model of celestial spheres by adding epicycles, deferents, equants and eccentrics
- Aristarchus (310-230 BC), later advocated by Seleucus of Seleucia (190 - 150 BC), hypothesized a heliocentric model with rotating earth. Stars were proposed to be distant suns which explained their fixed position (lack of stellar parallax).
- The retrograde motion of Mars was the impetus behind many of the revisions of the cosmological model. A Nice explanation of the how the retrograde motion of Mars explains many of these ancient models can be found HERE (Craig McConnell, Cal State Fullerton) or HERE.

Ptolemaic Model: Epicycle, Deferent, Equant, Eccentric

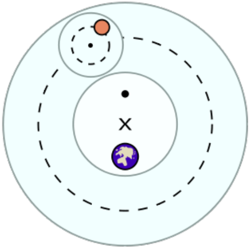




Concentric Circles:

- Daily Motion
- Yearly Motion
- Earth Doesn't rotate

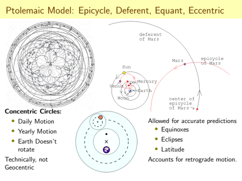
Technically, not Geocentric



Allowed for accurate predictions

- Equinoxes
- Eclipses
- Latitude

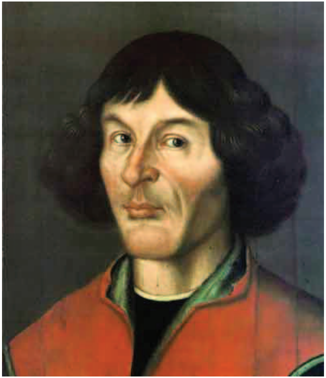
Accounts for retrograde motion.

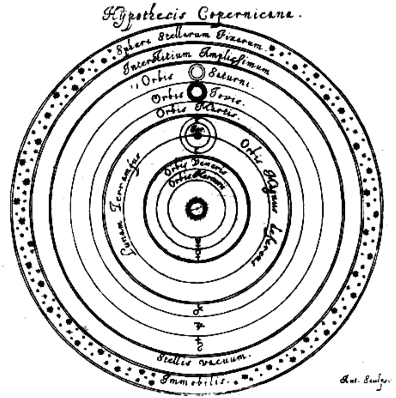


- Figure 3 shows a planet on an epicycle (smaller dashed circle), a deferent (larger dashed circle), the eccentric (X) and an equant (larger black dot)
- In the Hipparchian system the epicycle rotated and revolved along the deferent with uniform motion. However, Ptolemy found that he could not reconcile that with the Babylonian observational data available to him. The angular rate at which the epicycle traveled was not constant unless he measured it from another point which he called the equant. It was the angular rate at which the deferent moved around the point midway between the equant and the Earth (the eccentric) that was constant; the epicycle center swept out equal angles over equal times only when viewed from the equant. It was the use of equants to decouple uniform motion from the center of the circular deferents that distinguished the Ptolemaic system. [Cited]
- The use of equant and eccentric, however, complicates the motion of moon and sun - See Detailed Explanation.
- Don't confuse with Ptolemy the greek general and later pharoah of egypt.
- Epicycles may have been based on the work of Appolonius of Perga (colleague of Hipparchus)

Copernican Fix (1473-1543)

Increasing accuracy of observation made model of Ptolemy obsolete





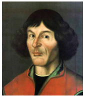
1. Swap earth/sun. 2. Earth is spinning. 3. Moon still orbits earth

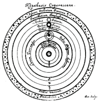
Question: Would Ptolemy have won without the moon? Confusing!!!

Developing the model

Copernican Fix (1473-1543)

Increasing accuracy of observation made model of Ptolemy obsolete

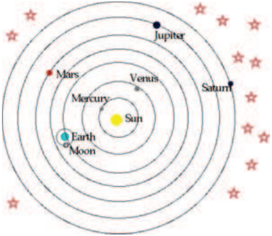




1. Swap earth/sun. 2. Earth is spinning. 3. Moon still orbits earth

Question: Would Ptolemy have won without the moon? Confusing!!!

- The moon is so close, there is no way it could be orbiting anything other than the earth.
- The moon always shows the same side, meaning it probably doesn't rotate about its axis.



Positives

- Aesthetically Appealing
  - Epicycles are much smaller
  - Less movement/rotation
- Intuitively appealing

Negatives

- No physical Explanation
  - Relies on Metaphysics, not physics
- No proof
- No empirical validation
- Still assumes circular orbits at constant speed
  - Still requires Epicycles, albeit smaller ones

Developing the model

Galileo Galilei (1564-1642)

The Ptolemaic Model was **"Disproven"** by the observations of Galileo

- Built the first decent telescope
- Observed the moons of Jupiter.
  - Showed that planets could orbit other planets
- Observed the phases of venus.
  - Death blow for Ptolemy's model
- An incorrect theory of tides.
- Imprisoned by church.

Developing the model

Galileo Galilei (1564-1642)

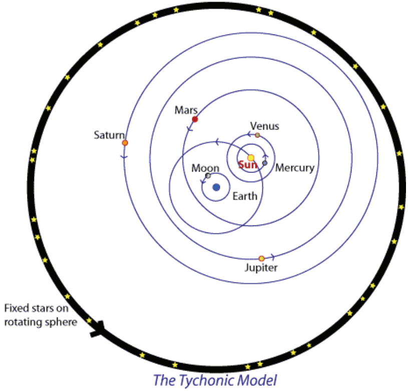
The Ptolemaic Model was **"Disproven"** by the observations of Galileo

- Built the first decent telescope
- Observed the moons of Jupiter.
  - Showed that planets could orbit other planets
- Observed the phases of venus.
  - Death blow for Ptolemy's model
- An incorrect theory of tides.
- Imprisoned by church.

- Observed motion of sunspots, showing that the sun is rotating.
- Observing the phases of Venus first establishes definitively that the planets are individual worlds and earth is one among many. (Sidereus Nuncius - The starry messenger/The message of the stars)
- Proposed that tides are caused by rotation about the sun.
- Significant advancement in kinematics and strength of materials
- The idea that forces cause motion is beginning to appear (causes the tides)
- Wrote "Il Saggiotore" (The Assayer) "The book of nature is writ in the language of mathematics, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering around in a dark labyrinth"
  - Gallileo was one of the great Italian writers.
  - Works are written in the form of disputation (dialogue/dialectic).
  - Annoyed the church and was summoned by the inquisition (by casting the pope as Simplicio). Was placed under house arrest for remainder of his life.
- Focus on not why, but how things work (using mathematics)
- Develop the first notion of relativity using a thought experiment, which was the inspiration for Einstein's generalized version or relativity.

Developing the model - The Tychonic System

Tycho Brahe (1546-1601)



Tycho Brahe was a man with

- A fake nose (silver-gold alloy)
- A colorful personality
- A very bad model

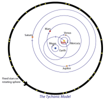
In the Tychonic system, the sun and stars orbit the earth. Everything else orbits the sun.

Developing the model - The Tychonic System

Tycho Brahe (1546-1601)

Tycho Brahe was a man with

- A fake nose (silver-gold alloy)
- A colorful personality
- A very bad model



The Tychonic Model

In the Tychonic system, the sun and stars orbit the earth. Everything else orbits the sun.

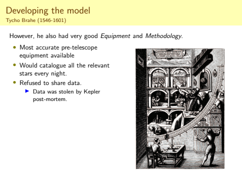
- Observations were around 5x more accurate than best available (Inspired by errors in the prediction of the Conjunction of Jupiter and Saturn). Given a private island for astronomy. When aboveground facilities were not sufficiently stable, built an underground observatory. (Tycho consitutes 1% of total Danish crown revenue in 1580s.)
- Still could not measure stellar parallax. (Not measured until 1838 by Bessel)
  – However, he could use parallax to measure the distance to comets.
- Died of a burst bladder.
- During a party thrown by his professor, lost his nose in a duel as a student at U. Rostock over who was a better mathematician.
- Had a tame elk (or moose), which he used to take to parties. Unfortunately, at one party, it drank too much beer, fell down a set of stairs and died.
- The Tychonic system was the main competitor with the Copernican system for some time (Ptolemaic system was discredited by Galileo). The inclusion of a stationary earth seemed intuitive and explained the lack of stellar parallax. Stellar parallax is the perceived relative motion of the stars due to the changing position of the earth.
- There is no observable difference between Tychonic and Copernican models. Simple change of coordinate systems.
- In the Tychonic system, the earth does not rotate. However, a student of Tycho's proposed adjusting Tycho's model to allow the earth to rotate.
- His father died of pneumonia after diving into the Copenhagen canal to rescue the drunken king, who had fallen in after a party.

Developing the model

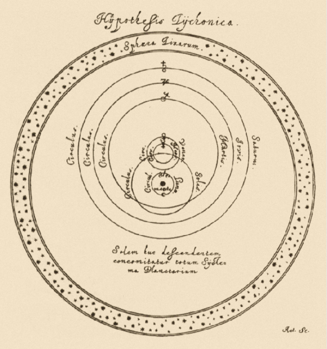
Tycho Brahe (1546-1601)

However, he also had very good *Equipment* and *Methodology*.

- Most accurate pre-telescope equipment available
- Would catalogue all the relevant stars every night.
- Refused to share data.
  - Data was stolen by Kepler post-mortem.



- Kepler worked with him the last year of his life, despite disagreement over his model.
- Tycho forced Kepler to accuse a rival (Rothmann) of plagiarising Tycho.



Developing the model

Johannes Kepler (1571-1630)

- Contemporary with Sir Francis Bacon (1561-1626), father of empirical science.
- Became an assistant to Tycho in order to get access to data.
    - Rudolphine Tables.
- Primarily observed the motion of Mars
- Formulated experimentally the three laws of planetary motion
    - No derivation.
- Postulated that earth exhibits a central force.
- A correct theory of tides.
- Ignored by Galileo, Descartes

Developing the model

Johannes Kepler (1571-1630)

- Contemporary with Sir Francis Bacon (1561-1626), father of empirical science.
- Became an assistant to Tycho in order to get access to data.
  - Rudolphine Tables.
- Primarily observed the motion of Mars
- Formulated experimentally the three laws of planetary motion
  - No derivation.
- Postulated that earth exhibits a central force.
- A correct theory of tides.
- Ignored by Galileo, Descartes

- Kepler was profoundly religious, almost reminiscent of the pythagoreans.
  - Tried to find the music that the planets make using geometric relationships
  - while teaching in Graz, **demonstrating the periodic conjunction of Saturn and Jupiter** in the zodiac: he realized that regular polygons bound one inscribed and one circumscribed circle at definite ratios, which, he reasoned, might be the geometrical basis of the universe. After failing to find a unique arrangement of polygons that fit known astronomical observations (even with extra planets added to the system), Kepler began experimenting with 3-dimensional polyhedra. He found that each of the five Platonic solids could be inscribed and circumscribed by spherical orbs; nesting these solids, each encased in a sphere, within one another would produce six layers, corresponding to the six known planets—Mercury, Venus, Earth, Mars, Jupiter, and Saturn. By ordering the solids selectively—octahedron, icosahedron, dodecahedron, tetrahedron, cube—Kepler found that the spheres could be placed at intervals corresponding to the relative sizes of each planet's path, assuming the planets circle the Sun. - The Mysterium (Wikipedia).

Kepler's Model of Planetary Motion

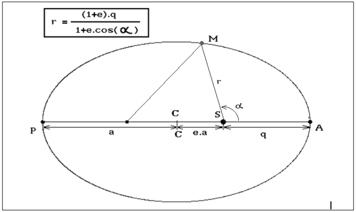
First Law of Planetary Motion

Still a model based on Geometry

- No understanding of forces and inertia
- No differential equations

Law 1: Planets move in elliptic orbits with one focus at the planet they orbit.

- The ellipse is a well-understood mathematical concept from geometry
- There are several well-studied parameters of the ellipse
  - a - semi-major axis
  - b - semi-minor axis
  - e - eccentricity

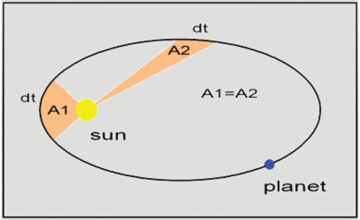


Doc.4

- Appolonius of Perga defined ellipses in his Book I of Conics in around 200BC.

Law 2: Planets sweep out equal areas of the ellipse in equal time.

- First model to posit that planets slow down and speed up.
- $\dot{A}$ is constant for each planet (A is area)
- Allows for quantitative predictions of locations and time.
  - Allowed him to formulate Rudolphine tables.



$$\frac{dA}{dt} = \frac{\|\vec{h}\|}{2} = \frac{\sqrt{\mu a(1-e^2)}}{2} = constant$$

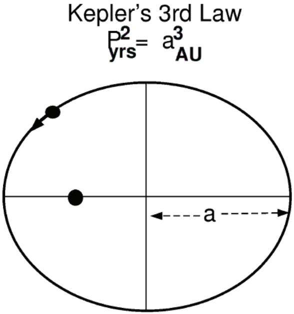
A precursor to a differential equation model.

Law 3: The square of the period of the orbit is proportional to the cube of the semi-major axis.

- A simple corollary of the second law?
  - Second law applies to each orbit
  - Area of ellipse:

$$\text{Area}_{ellipse} = a^2\sqrt{1-e^2}$$

- Third law implies the rate of sweep changes from orbit to orbit



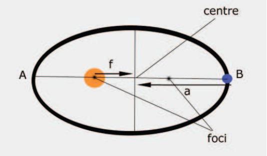
$$\frac{T^2}{a^3} = \frac{4\pi^2}{\mu} = constant$$

Developing the model

More on Kepler's model

Got almost everything right

- Postulated a central-force hypothesis
  - First theory based on physics.
  - Pre-Newton, so no three laws of motion, so no inertia.
- Created a correct explanation for tidal motion
- Made the most accurate predictions



Kepler's model was not initially accepted

- ignored by big names (Galileo, Descartes, etc.)
  - Galileo had his own tides model
- Used but not believed (Tychonic model still dominates)

Still no derivation from physical principles. Must wait almost 60 years for an explanation.

Developing the model

Isaac Newton (1643-1727)

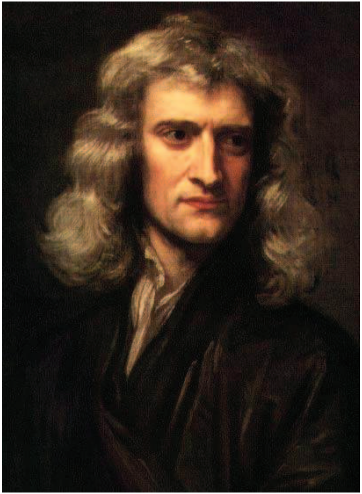
- Most influential person in history?

A quantitative model for motion

- Discovered differential equations
- Discovered modeling nature using equations
  - Now we can use differential equations to model Everything.

Newton used differential equations to model

- Force
- Velocity
- Acceleration
- Inertia
- Gravity



Developing the model

Isaac Newton (1643-1727)

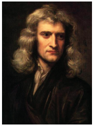
- Most influential person in history?

A quantitative model for motion

- Discovered differential equations
- Discovered modeling nature using equations
  - Now we can use differential equations to model Everything.

Newton used differential equations to model

- Force
- Velocity
- Acceleration
- Inertia
- Gravity



- Involved in a bitter priority dispute with Leibnitz over integral and differential Calculus.
- "If I have seen further it is by standing on the shoulders of giants."
- Despite modesty, it was reported that Newton was actually a very bad/boring teacher - Isaac Newton's assistant Humphrey Newton (not a relation) later reported that "so few went to hear him [that] oftentimes he did [. . .] for want of Hearers, read to ye Walls"
- Hero of the enlightenment. English poet Alexander Pope wrote the famous epitaph: "Nature and nature's laws lay hid in night; God said "Let Newton be" and all was light."
- The great work, Philosophiae naturalis principia mathematica, does not actually use calculus notation, instead relying on very tedious geometric equivalents.
- In later life, focused on biblical scholarship. His personal religious beliefs, however, were unique and considered heretical.
- Became warden of the Royal mint, where he went undercover in bars and taverns to find forgers, of which 28 were convicted.
- On deathbed, claimed his most significant accomplishment was remaining celebate.
- in 1693, accused John Locke (and Samuel Pepys) of sending women to seduce him.

Three Laws of Motion:

Law 1 Every body remains at rest or in uniform motion unless acted upon by an unbalanced external force. (Already studied by Galileo, others)

Law 2 A body of mass m, subject to force F, undergoes an acceleration a where

$$\vec{F} = m\vec{a} = m\ddot{\vec{r}}$$

Law 3 The mutual forces of action and reaction between two bodies are equal, opposite and collinear. (In popular culture: Every action creates an equal and opposite reaction.)

Law of Universal Gravitation:

- All bodies exert a force on all others
  - Proportional to mass
  - Inversely proportional to the square of the distance

$$F = G\frac{m_1 m_2}{r^2}$$

- From this, Newton derived Kepler's laws of planetary motion. (Deduction)
  - Or perhaps derived universal gravitation from Kepler's laws. (Induction)

Developing the model

Philosophiæ Naturalis Principia Mathematica

Three Laws of Motion:

- **Law 1** Every body remains at rest or in uniform motion unless acted upon by an unbalanced external force. (Already studied by Galileo, others)
- **Law 2** A body of mass m, subject to force F, undergoes an acceleration a where
$$\vec{F} = m\vec{a} = m\ddot{\vec{r}}$$
- **Law 3** The mutual forces of action and reaction between two bodies are equal, opposite and collinear. (In popular culture: Every action creates an equal and opposite reaction.)

Law of Universal Gravitation:

- All bodies exert a force on all others
  - Proportional to mass
  - Inversely proportional to the square of the distance
$$F = G\frac{m_1 m_2}{r^2}$$
- From this, Newton derived Kepler's laws of planetary motion. (Deduction)
  - Or perhaps derived universal gravitation from Kepler's laws. (Induction)

- "To every action there is always opposed an equal reaction" is an actual quote from Newton
- "Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it" is also a quote from Newton.

Also discovered

- Refracting Telescope
- Integral calculus
- Infinite sequences and series
  - Binomial theorem
- Partial Differential Equations
- Model for wave motion
  - Theory of Color (and optics)
  - Speed of Sound
- Algorithms for solving nonlinear equations
  - Newton's method is still the most common optimization algorithm.
  - We will use it in this class.

The model

The Two-Body Problem

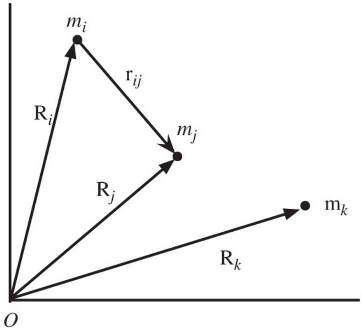
The force on mass 1 due to mass 2 is

$$m_1\ddot{\vec{R}}_1 = \vec{F}_1 = G\frac{m_1 m_2}{\|\vec{r}_{12}\|^3}\vec{r}_{12}$$

where we denote $\vec{r}_{12} = \vec{R}_2 - \vec{R}_1$. Clearly $\vec{r}_{12} = -\vec{r}_{21}$. The Force on mass 2 due to mass 1 is

$$m_2\ddot{\vec{R}}_2 = \vec{F}_2 = G\frac{m_2 m_1}{\|\vec{r}_{21}\|^3}\vec{r}_{21}.$$

The problem is a nonlinear coupled ODE with 6 degrees of freedom.



Solution: Consider relative motion (only $\vec{r}_{12}$)

$$\ddot{\vec{r}}_{12} = -\frac{G(m_1+m_2)}{\|\vec{r}_{12}\|^3}\vec{r}_{12}$$

This is our model.

Orbital Mechanics

In the first part of this class, we study:

Definition 1.

Orbital Mechanics is the study of motion about a center of mass.

More generally, the field is

Definition 2.

Celestial Mechanics is the study of heavenly bodies

- Also includes
  - Black holes
  - Dark matter
  - Big Bang Theory
  - Relativistic mechanics

We will stick to orbits.

Conclusion

In this Lecture, you learned:

History

- Orbital mechanics is old
- Much of science and mathematics was developed to create the model we use in this class
  - Physical Modeling
  - Models using differential equations
  - Empirical Science

The Model

- Universal Gravitation
- Two-Body Problem
  - 3DOF equations of relative motion

Next Lecture: The two-body problem

Universal Invariants

- Angular Momentum
- Linear Momentum

N-body Problem

- Introduction
- Invariants

Derivation of Kepler's Laws

- Kepler's First Law