MAE 598: LMIs in Optimal and Robust Control

Syllabus

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Meeting Time: M (virtual) W 4:35-5:50 (We may occasionally have Monday meetings - check Canvas for announcements) in ECGG335 (location may change if in-person instruction becomes unworkable)
Office Hours: F 4:00-5:00PM (online only)
Outside Office Hours: Appointment by email

Motivation The recent introduction of Linear Matrix Inequality (LMI) methods in control has dramatically expanded the types and complexity of the systems we can control. For example, consider the problems: Gain Scheduling for Missile Attitude Control (A switched system); Control of Robots over Noisy Communication Channels (a sampled-data systems); Remote Control of Spacecraft Attitude (a delayed system); Behavioural Therapy (A system with binary inputs); or self-driving vehicles (a case of decentralized control). None of these systems can be studied using classical root-locus or PID control methods. Rather, advances in these fields have been made possible through the increased power and flexibility created by the LMI (optimization-based) approach to control.

Course Structure This class will include both in-person and online components. Two weekly lectures will be posted on the canvas site by the typical Monday meeting time. There will be no in-person or virtual Monday meeting times. All students should then view both lectures prior to the Wednesday lecture. Attendance at the Wednesday lecture is MANDATORY and students will be strongly penalized for failure to attend. At the Wednesday Lecture, we will employ the disputation lecture format. All students should be prepared to answer the questions posed by the instructor and to support their response using material from the lecture slides. Students should also be prepared to rebut or dispute answers given by other students. Students participating online and from abroad will also we required to attend this lecture and to answer questions and engage in disputation. This structure is subject to change as becomes necessary due to 2020 COVID pandemic.

Course Objectives To be able to use LMI solvers to synthesize optimal or suboptimal controllers and estimators for multiple classes of state-space systems.

Learning Outcomes Upon completion of this course, students will be able to:

1. Recognize the three parts of an optimization problem: Variables, Objective, Constraints
2. Categorize an optimization problem as convex or non-convex
3. Categorize a convex optimization problem as semidefinite programming, linear programming, second-order cone programming, etc.
4. Recognize the difference between tracking and regulator frameworks
5. Construct the 9-matrix representation of an optimal control problem
6. Construct an H_∞-optimal dynamic output feedback controller using LMI solvers

7. Implement and Test a dynamic output feedback controller on a state-space system
8. Construct the LFT representation of a robust control problem
9. Construct a robust state-feedback controller using LMI solvers

Grading Policies There will be approximately 5 homework assignments which cumulatively will be the basis for 30% of the course grade. Problem sets will be given on a bi-weekly basis. A midterm will be given for 30% of the grade. The final project or final exam will be for 40% of the grade.

Absence Policy Attendance at all in-person lectures is required. Exceptions include excused absences related to university sanctioned events/activities that are in accord with ACD 304–02, "Missed Classes Due to University-Sanctioned Activities" and excused absences related to religious observances/practices that are in accord with ACD 304–04, "Accommodation for Religious Practices". In general, late homework will not be graded and missing examinations cannot be retaken. Exceptions include excused absences related to university sanctioned events/activities that are in accord with ACD 304–02, "Missed Classes Due to University-Sanctioned Activities" and excused absences related to religious observances/practices that are in accord with ACD 304–04, "Accommodation for Religious Practices"

Content In this course, we provide an overview of LMIs and their many applications in Modern Control Theory. In contrast to other courses in control, our subject matter will be more focused on the variety of applications for LMIs and less on theory. Nonetheless, there is a significant theoretic component and students should expect to perform rigorous mathematical proofs.

The primary text for the class is "LMIs in Control Systems: Analysis, Design and Applications" by G.-R. Duan and H.-H. Yu. This text is not organized in the same way as the course, however. The second text we will use is "Linear Matrix Inequalities in System and Control Theory" by S. Boyd. This second text is freely available online from the author and may be found at

https://web.stanford.edu/ boyd/lmibook/lmibook.pdf.

A more theoretical treatment of some of the material can be found in "A Course in Robust Control Theory: A Convex Approach" by G. Dullerud and F. Paganini. Be warned, however, that many students find this text to be difficult to follow.

I will assume that students are familiar with the basics of state-space systems or are capable to learning about them very quickly. For an accessible text on State-Space methods, I recommend "Linear State-Space Control Systems" by D. A. Lawrence. If you have limited exposure to state-space, it might be a good idea to review the first few chapters of this book.

Schedule Class meets on Monday and Wednesday from 4:35-5:50 in room LSA 119. Assignments will be given approximately bi-weekly. There will be a midterm and a final project/exam.

Lecture Format Lectures will utilize a constantly evolving set of LaTeX slides. A previous incarnation of these slides is available on my website `http://control.asu.edu` under the classes heading - MAE 598, as well as on blackboard. As the slides are modified, they will be updated on blackboard and ultimately on my website.

Blackboard Lecture Notes will be posted on Blackboard, along with all assignments and supplementary material. Grades will also be posted on blackboard.

Prerequisites There is no prerequisite for this class. However, it is assumed that all students have some background in controls and linear algebra. Access to Matlab is required, including the robust control toolbox - available in the ASU computing laboratories.

Final Project In lieu of a final exam, this class will have a final project. This project is expected to be development of a new LMI. Part of the project will be documentation of this LMI on the course Wikibook. This project may be based on your own research or may be on an entirely new topic. The project should leverage LMI methods to solve a problem which has not already been solved. Some problem suggestions are listed at the end of this syllabus. You may work in groups of up to 2 people. However, if you work in a group of 2, I will expect double the results as of a group of 1. All project topics should be submitted to me in advance. If you do not wish to do a project, you may take a final exam instead.

Classroom Behavior The use of laptops, iphones, ipads, or other personal electronic devices is prohibited during class unless explicitly allowed by the instructor.

Violence See the Student Services Manual, SSM 104–02, "Handling Disruptive, Threatening, or Violent Individuals on Campus"

Disabilities A reminder to students that when requesting accommodation for a disability, they must be registered with the Disability Resource Center (DRC) and submit appropriate documentation from the DRC

Academic Integrity Collaboration on exams will result in an E for the course. Copying of homework or duplication of material found online will result in a "0" on the homework and a referral to the ASU office of academic integrity. **Reminder:** Two referrals to the office of academic integrity is grounds for expulsion from the university. If in doubt about a specific case, ask me.

Plagiarism Plagiarism applies to homework, numerical code, and the final project. In all cases, it refers to the use of outside material without attribution. In the case of code, it can mean re-arrangement of lines, renaming of variables or other cosmetic changes. In the case of the final project, it includes "rephrasing" of published or unpublished material. This occasionally includes proofs. The penalty for plagiarism on exams is an E for the course. The penalty for plagiarism on homework is as defined in the paragraph on Academic Integrity.

References Aside from the text, there are several excellent optional sources which may be consulted. Although not directly required for the course, students are encouraged to browse the following references.

The following is an introduction to classical control and state-space theory.

- Franklin, Powell and Enami. "Feedback Control of Dynamical Systems", Addison-Wiley, 1994.

The following are references for LMI methods in control.

- Zhou, Doyle and Glover. "Robust and Optimal Control", Prentice Hall, 1996.
- Boyd, El Ghaoui, Feron and Balakrishnan. "Linear Matrix Inequalities in Systems and Control Theory", SIAM, 1994.

The following is a thorough reference on matrix analysis.

- Horn and Johnson. "Matrix Analysis", Cambridge University Press, 1985.

The following is a clearly written text on mathematical analysis.

- Marsden and Hoffman. "Elementary Classical Analysis", Freeman 1993.

Assignments:

- Week 1-3 **Read:** Duan, chapter 1, 3, **Assignment:** HW 1
- Week 4-5 **Read:** Duan, chapter 4.1,4.2, 6.1, 6.2, 7.1, 7.2, 9.1.1, **Assignment:** HW 2
- Week 6-7 **Read:** Duan, chapter 8.1,8.2,8.4, **Assignment:** HW 3
- Week 8-9 **Midterm Examination**
- Week 10-11 **Read:** Duan, chapters 7.3, 8.3, 11, **Assignment:** HW 3
- Week 12-13 **Read:** Course Notes on SOS/PDEs/Switched Systems
- Week 14-15 **Final Examination**

Outline: The following is a rough outline of the set of lectures which will be presented.

Lecture 1 - Introduction
Lecture 3 - Introduction to Optimization
Lecture 4 - Introduction to Positive Matrices and LMIs
Lecture 5 - Our First LMI
Lecture 2 - LMIs for Stability, Controllability and Observability.
Lecture 6 - The Optimal Control Framework
Lecture 7 - An LMI for Full-State Feedback Controller Synthesis
Lecture 8 - An LMI for H_2-Optimal Full-State Feedback Control (LQR)
Lecture 9 - The H_∞ norm
Lecture 10 - An LMI for H_∞-Optimal Full-State Feedback Control
Lecture 11 - The Luenberger Observer and The Kalman Filter
Lecture 12 - An LMI for Output-Feedback Stabilization
Lecture 13 - An LMI for H_∞-Optimal Output Feedback Control
Lecture 14 - Systems with Uncertainty and the Structured Singular Value
Lecture 15 - An LMI for Robust Stability Analysis
Lecture 16 - The D-K iteration
Lecture 17 - Switched Systems
Lecture 18 - An LMI for Control of Systems with Switching
Lecture 19 - Delayed Systems
Lecture 20 - An LMI for Stability of Systems with Delay
Lecture 21 - Nonlinear Systems
Lecture 22 - An LMI for Stability of Nonlinear Systems

Examples of Proposed Project Topics:

- Gain Scheduling for Missile Attitude Control (Switched Systems)
- Control of Robots over Noisy Communication Channels (Sampled-Data Systems)
- Spacecraft Attitude Control with delayed communication (Delay Systems)
- Social Cognitive Therapy using Discrete Inputs (Mixed-Integer Control)
- Self-Driving Vehicles (Decentralized Control)